228B Homework 1

due Thurs, Feb. 1

1. solve $u_t = u_{xx}$, $u(x,0) = x(1-x)$, $u(0,t) = u(1,t) = 0$
on the finite interval $0 \le x \le 1$ to time $T = 1$

(a) set $h = k = 1/10$ → show instability

(b) $h = 1/10$, $k = 1/200$ → show stability

(c) try the trick with $\nu = 1/6$
see if you can see a higher order of convergence
i.e. compare convergence rates for $\nu = \frac{1}{2}$ and $\nu = \frac{1}{6}$
(you'll have to figure out how to estimate
the order at which your solution is converging)

2. consider $u_t = \alpha u_{xx}$, $u(x,0) = g(x)$
on an unbounded domain. what does ν have
to be for stability?

3. For $x \in \mathbb{R}^n$, define $\|x\|_1 = \sum_{i=1}^{n} |x_i|$, $\|x\|_\infty = \max_{1 \le i \le n} |x_i|$

Let A be an $n \times n$ matrix. Prove that

$$\|A\|_1 = \max_j \sum_i |A_{ij}|$$ max absolute column sum

$$\|A\|_\infty = \max_i \sum_j |A_{ij}|$$ max absolute row sum