Homework 6
due Thursday, Nov. 8

(1-3) Compute the stability funtion $R(z)$ for each of the following methods and determine which are A-stable and L-stable.

SDIRK:
$$\begin{array}{c|cc} \beta & \beta & 0 \\ 1-\beta & 1-2\beta & \beta \\ \hline & 1/2 & 1/2 \end{array} \qquad \beta = \frac{3\pm\sqrt{3}}{6},\ \frac{2\pm\sqrt{2}}{2},\ 1,\ \frac{1}{4}$$

Radau IIa:
$$\begin{array}{c|cc} 1/3 & 5/12 & -1/12 \\ 1 & 3/4 & 1/4 \\ \hline & 3/4 & 1/4 \end{array},$$

Hammer-Hollingsworth:
$$\begin{array}{c|cc} \frac{1}{2}-\frac{\sqrt{3}}{6} & \frac{1}{4} & \frac{1}{4}-\frac{\sqrt{3}}{6} \\ \frac{1}{2}+\frac{\sqrt{3}}{6} & \frac{1}{4}+\frac{\sqrt{3}}{6} & \frac{1}{4} \\ \hline & 1/2 & 1/2 \end{array}$$

(4-6) Solve $y' = \lambda(y - \sin t) + \cos t$, $y(0) = 0$ for $0 \le t \le 1$ with $h = 1/N$, $\lambda = -N^2$ for several values of N using each of the methods above. Determine the rate at which the numerical solution y_N is converging to the exact solution $y(1)$ as $h \to 0$. (Note: $\lambda = -\frac{1}{h^2} \to \infty$ as $h \to 0$, i.e. the problem is changing as we refine the mesh). Some of the schemes will not converge at all; omit those schemes.