Math 228A, Fall 2007, Wilkening

Homework 5
due Thursday, Oct. 25

(1) Compute $\alpha(\phi)$ for each $\phi \in S_6$ and use the Faá di Bruno formula to evaluate

$$(a)\ \frac{d^5}{dt^5}\exp(\sin t), \qquad (b)\ \frac{d^5}{dx^5}\frac{1}{g(x)}.$$

Note: in Lecture 13, the recursive definition I gave for α is wrong. (I erased it from the online version of the notes). Your task is to figure out how to count the number of ways of labeling trees (of order 6) that have no ramifications (branching nodes) except at the root. Also note that in 1d, our multilinear derivative operators are just multiplication by the derivative, e.g. $D^2 f(y)(u,v) = f''(y)uv$.

(2) Verify that the SDIRK method

$$\begin{array}{c|cc} \beta & \beta & 0 \\ 1-\beta & 1-2\beta & \beta \\ \hline & 1/2 & 1/2 \end{array} \qquad \beta = \frac{3 \pm \sqrt{3}}{6}$$

is third order. (Write down the order conditions corresponding to all graphs of order ≤ 3 and check that this scheme satisfies them).

(3) Find the coefficients b_i of the 9 stage explicit Runge-Kutta method with

$$a_{ij} = \begin{cases} 1 & i = j+1 \\ 0 & \text{otherwise} \end{cases}$$

such that the method is 9th order when applied to the linear, constant coefficient ODE $y' = By$, where B is a $d \times d$ constant matrix. Hint: which order conditions are automatically satisfied when $f(y) = By$ depends linearly on y? How can you organize the remaining order conditions into a linear system for the unknowns b_i?

(4) Use your method in (3) to evaluate the matrix exponential

$$\exp\begin{pmatrix} 0 & -3 & 5 \\ -1 & -6 & 11 \\ 0 & -4 & 7 \end{pmatrix}$$

to as many correct digits as you can get in double precision arithmetic.