Math 228A, Fall 2007, Wilkening

Homework 1
due Tuesday, Sep. 11

(1) Consider the equation for a pendulum, $\theta''(t) + \sin(\theta(t)) = 0$.
(a) Choose initial conditions and calculate one full swing by Euler's method
(b) Find the frequency of your pendulum. (Explain how you computed the frequency and why you did it this way)
(c) What is the theoretical error bound? (Hint: what is is the Lipschitz constant for the sine function? What about for the right hand side of the first order system? How can you use energy conservation to obtain a bound on $\|y''(t)\|$? How can you turn an error estimate for y into an estimate for the period of one swing?)

(2) The function

$$S(x) = \int_0^x \sqrt{1 + \sin^2 t}\, dt, \qquad (0 \le x \le 2\pi) \tag{1}$$

is the arclength function for the curve $y = \cos x$.
(a) find the length L of this curve using the trapezoid rule to evaluate the integral. Observe that L converges exponentially fast as the number of integration points is increased. (L is not the Lipschitz constant in this problem).
(b) Suppose we want to reparametrize the curve via arclength. Find the differential equation for $x(s)$ such that $S(x(s)) = s$ for all s.
(c) Code up Euler's method and the trapezoidal rule method to solve this ODE. (use Newton's method to solve the implicit equation in the latter case). Evaluate the numerical solution at L and compare to the exact answer 2π. Explain why Euler's method is working as well as the trapezoidal rule method. (Hint: think about the variational equation satisfied by the error). Now compare the two methods for evaluating $x(s)$ for other values of s between 0 and L to confirm that Euler's method is only converging at first order over most of the interval $0 \le s \le L$ while the trapezoidal rule method is second order throughout.

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(This week:
Fri: 10-11
Next week:
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