homework 5 due Thurs 10/26

1. use graph theory to compute the order conditions through 4th order (8 conditions in all, one for each tree in the handout)

$$\sum_{i=1}^{s} b_i A_i(\hat{t}) = \frac{1}{\gamma(\hat{t})}$$

2. Verify that the SDIRK method

β	β	0
$1-\beta$	$1-2\beta$	β
	$1/2$	$1/2$

$$\beta = \frac{3 \pm \sqrt{3}}{6}$$

is 3rd order.

3. Verify that the classical explicit RK method

0				
$1/2$	$1/2$			
$1/2$	0	$1/2$		
1	0	0	1	
	$1/6$	$2/6$	$2/6$	$1/6$

is 4th order.

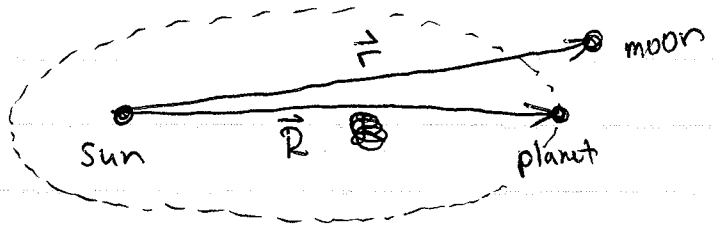
[redacted]

[redacted]

4. see reverse side

4. Let $\vec{R} = (X, Y)$, $\dot{\vec{R}} = (U, V)$

$\vec{r} = (x, y)$, $\dot{\vec{r}} = (u, v)$



Solve the ODE

$$\ddot{\vec{R}} = -\frac{\vec{R}}{\|\vec{R}\|^3} + \frac{\vec{r} - \vec{R}}{\|\vec{r} - \vec{R}\|^3} \cdot \frac{1}{8000}$$

$$\ddot{\vec{r}} = -\frac{\vec{r}}{\|\vec{r}\|^3} + \frac{\vec{R} - \vec{r}}{\|\vec{R} - \vec{r}\|^3} \cdot \frac{1}{100}$$

using the 4th order classical RK method and Richardson extrapolation with stepsize control

use $Atol = 10^{-6}$, $Rtol = 10^{-6}$

Try the initial conditions

$\vec{R}(0) = (5, 0)$ $\quad$ $\dot{\vec{R}}(0) = (0, 0.2)$

$\vec{r}(0) = (5.1, 0)$ $\quad$ $\dot{\vec{r}}(0) = (0, 0.3)$

plot the trajectory of the planet and moon over one year.