

Homework 1
due Thursday, Sep. 7

(1) Suppose $y : [a, b] \rightarrow \mathbb{R}^d$ is a continuous function. Show that $\left\| \int_a^b y(t) dt \right\| \leq \int_a^b \|y(t)\| dt$, where $\|\cdot\|$ is any norm on \mathbb{R}^d . Hint: one reasonable approach is to break the integral into a Riemann sum and use the triangle inequality:

$$\int_a^b y(t) dt = \lim_{n \rightarrow \infty} \sum_{k=0}^{n-1} y(t_{kn}) h_n, \quad \left(t_{kn} = a + \frac{k}{n}(b-a), h_n = \frac{b-a}{n} \right).$$

Finish this argument, justifying your steps.

- (2) Consider the equation for a pendulum, $y''(t) + \sin(y(t)) = 0$.
- (a) Choose initial conditions and calculate one full swing by Euler's method
 - (b) Find the frequency of your pendulum
 - (c) What is the theoretical error bound?
 - (d) What do you think the error really is?