Homework 4
due Fri, April 7

1. (a) Let $I = [a, b]$ be a finite interval. Show that the space $C(I, \mathbb{R}^n)$ of continuous functions from I into $\mathbb{R}^n$ is a Banach space with the uniform norm
$$\|u\|_\infty = \sup_{t\in I} |u(t)|.$$
(Show that this is a norm and that $C(I, \mathbb{R}^n)$ is complete.)

   (b) Let $r > 0$, $x_0 \in \mathbb{R}^n$, and $B = \{x \in \mathbb{R}^n \ : \ |x - x_0| \leq r\}$. Show that $C(I, B)$ is a closed subset of $C(I, \mathbb{R}^n)$, and is therefore a complete metric space.
2. p. 206 #1.1.
3. p. 219 #2.5
4. p. 219 #2.7.