

index

#

#, # definition of a limit , 31,64

A

absolute maximum and minimum , 199

absolute maximum and minimum values , 199

absolute value function , 5

absolutely convergent series, 446

acceleration , 282

Airy function , 457

Airy, Sir George , 457

alternating harmonic series , 444

alternating series , 442

Alternating Series Estimation Theorem , 445

Alternating Series Test , 443

antiderivative

 general vs. particular . 333

antidifferentiation formulas , 244

approximate integration , 336

approximating cylinder , 369

approximation

 by Taylor polynomials . 476

approximation, linear , 133

arc length , 382,385

arc length formula , 383

Archimedes , 394

area

 cross-sectional . 372

Area Problem , 253

arrow diagram for a function , 2

astroid , 126,141

asymptote(s)

 slant . 226

average cost function , 235

average rate of change

 units for . 80

average speed of molecules , 356

average value of a function , 291

average velocity , 76

B

Barrow, Isaac , 286

base

 change of . 158

base of a cylinder , 366

Bernoulli, James , 402

Bernoulli, John , 464

Bessel function , 453,457

Bessel, Friedrich , 453

binomial coefficient , 470

binomial series , 469,470,471

blood flow , 137

bounded sequence , 418

Boyle's Law , 5

bullet-nose curve , 120

C

cable, hanging , 182

calculator, graphing , 421

calculus, differential vs. integral , 286

cancellation equations , 151

Cantor set , 433

Cantor, Georg , 433

cardioid , 126

carrying capacity , 405

CAS

 See computer algebra system.

catenary , 182

Cauchy, Augustin-Louis , 37

Cavalieri's Principle , 376

Cavalieri, Bonaventura , 343

center of gravity , 394

center of mass , 394,395,397

centroid

 of a plane region . 395,397

Chain Rule , 113,114,116

change of base in a logarithm , 158

change of variables:

 in an integral . 295

charge, electric , 161

circle

 area of . 319

circular cylinder , 366

Closed Interval Method , 203

coefficient

 of a power series . 452

combinations of functions , 18

common ratio , 426

comparison properties of the integral , 273

Comparison Test , 438

comparison tests for series , 437

Comparison Theorem for integrals , 354

Completeness Axiom , 423

composite function

 derivative of . 113

composition of functions , 19

compound interest , 173,174,194

computer algebra system

 for integration . 461

concavity , 215

Concavity Test , 216

constant function

 derivative of . 95

Constant Multiple Law of limits , 36

Constant Multiple Rule , 98

continuity

 of inverse functions . 153

continuous compounding of interest ,
173,174,194

continuous function , 46

convergence

 of a series . 426

convergent improper integral , 349,352

convergent sequence , 417

convergent series

 properties of . 430

cosine function

 power series for . 469,471

cost function , 232

Coulomb's Law , 226

critical number , 203

cross-section , 367

cross-sectional area , 372

cubic function , 13

current, electric , 323

curve sketching

 with technology . 224

curve(s)

 smooth . 382

cylinder , 366

cylindrical shell , 377

D

decay, law of natural , 168

decay, radioactive , 170

decomposition of a function , 20

decreasing function , 7

decreasing sequence , 418

definite integral

 Substitution Rule for . 298

definite integration

 by parts . 307,308

degree of a polynomial , 13

delta (#) notation , 78

demand function , 232

density

index

- mass vs. weight . 392
- dependent variable , 2
- derivative(s)
 - of a power series . 459
- Descartes, René , 201
- Devil's curve , 126
- Difference Law of limits , 36
- Difference Rule , 99
- differentiable function , 87
- differential , 135
- differential calculus , 286
- differential equation
 - solution of . 402
- differentiation
 - term by term . 459
- differentiation operator , 86
- Direct Substitution Property , 37
- direction field , 407
- discontinuity of a function , 46,47
- discontinuous integrand , 352
- disk method , 368,369,372
- displacement , 281
- Distance Problem , 260
- divergence
 - of a sequence . 417
- Divergence, Test for , 430
- divergent improper integral , 349,352
- divergent sequence , 417
- divergent series , 426
- division of power series , 472
- domain of a function , 2
- E**
- e (the number):
 - as a sum of an infinite series . 467
- electric charge , 161
- electric current , 323
- elementary function , 335
- elimination constant of a drug , 198
- end behavior of a function , 68
- endpoint extreme values , 203
- equation(s)
 - differential .
- equation(s)
 - differential
 - (see differential equation),
- error
 - in Taylor approximation . 477
- error bounds , 340,344
- error estimate
 - for alternating series . 445
- error function , 294
- escape velocity , 400
- Euclid , 37
- Eudoxus , 37
- Euler, Leonhard , 435,467
- Evaluation Theorem , 277
- even function , 221
- exponential decay , 168
- exponential function(s)
 - power series for . 467,471
- exponential graph , 145
- exponential growth , 168
- Exponents, Laws of , 145
- extreme value , 199
- Extreme Value Theorem , 200
- F**
- family of exponential functions , 145
- family of solutions , 403
- fat circle , 125
- Fermat's Principle , 236
- Fermat's Theorem , 201
- Fermat, Pierre , 201
- Fibonacci , 416,424
- Fibonacci sequence , 416,424,433
- First Derivative Test
 - for Absolute Extreme Values . 230
- first-order differential equation , 402
- fixed point of a function , 212
- floor function , 40
- flux , 137
- folium of Descartes , 122
- foot-pound (unit of work) , 389
- force
 - exerted by liquid . 391
- fractions, partial , 323
- Fresnel function , 289
- Fresnel, Augustin , 289
- function(s)
 - representation as a power series . 457
- Fundamental Theorem of Calculus , 286,288,290
- G**
- Galileo , 24,37
- Gauss, Karl Friedrich , 265
- geometric series , 426
- graph(s)
 - of a sequence . 417
- graphing device, used to graph:
 - sequence . 421
- gravitation law , 400
- Gravitational acceleration , 388
- greatest integer function , 40
- Gregory's series , 461
- Gregory, James , 461,464
- growth rate , 282
- growth, law of natural , 168
- H**
- half-life , 171
- harmonic series , 429,437
- Heaviside function , 23,29
- Heaviside, Oliver , 29
- Hecht, Eugene , 135,137
- higher derivative(s) , 90
- Hooke's Law , 390
- horizontal asymptote , 60
- Horizontal Line Test , 149
- Huygens, Christiaan , 402
- hydrostatic pressure and force , 391
- hyperbolic function(s)
 - inverse . 184
- I**
- I/D Test , 213
- ideal gas law , 113
- implicit differentiation , 121,122
- implicit function , 402
- improper integral
 - convergence/divergence of . 349,352
- increasing function , 7
- increasing sequence , 418
- Increasing/Decreasing Test , 213
- increment , 78
- indefinite integral(s)
 - table of . 279
- independent variable , 2
- indeterminate forms of limits
 - quotient . 188,189
- infinite discontinuity , 47
- infinite interval , 348
- infinite limit , 56,63
- infinite sequence
 - See sequence.

index

infinite series

See series.

inflection point , 216

initial condition , 403

initial-value problem , 403

instantaneous rate of change , 79

instantaneous velocity , 76

integral

trigonometric . 313

integral calculus , 286

Integral Test , 434,436

integrand

discontinuous . 352

integration

term by term . 459

interest compounded continuously ,
173,174,194

Intermediate Value Theorem , 52

interval of convergence , 454,455

inverse function(s)

steps of finding . 152

inverse hyperbolic functions

expressed in terms of logarithms . 185

inverse substitution in integration , 318

inverse trigonometric functions

derivatives of . 177,178,180

J

Joule (unit of work) , 389

jump discontinuity , 47

K

kampyle of Eudoxus , 126

Kondo, Shigeru , 467

L

l'Hospital's Rule , 189

l'Hospital, Marquis de , 189,195

Lagrange's form of the remainder term , 466

Lagrange, Joseph-Louis , 207,209

lamina , 395

law of gravitation , 400

law of natural growth or decay , 168,169

law of the lever , 394

laws of exponents , 145

laws of logarithms , 156

learning curve , 409

left-hand limit , 29

Leibniz notation , 265,289

Leibniz, Gottfried Wilhelm , 402

lemniscate , 126

length

of a curve . 382

libration point , 242

Limit Comparison Test , 439

Limit Laws

for sequences . 419

limit(s)

of a sequence . 417

limits of integration , 265

line

tangent . 75

linear approximation

applications of, to physics . 135

linear function , 11

linear model , 11

linearization , 133

liquid force , 392

local maximum and minimum , 200

local maximum and minimum values , 199

logarithm(s)

notation for . 156

logarithmic differentiation , 164

logarithmic function(s)

properties of . 156

logistic differential equation , 405

logistic growth , 405

M

machine diagram of a function , 2

Maclaurin series

table of . 471

Maclaurin, Colin , 464

marginal cost function , 232

marginal profit function , 233

marginal propensity to consume or
save . 432

marginal revenue function , 232

mass density , 392

mass, center of , 394,395,397

mathematical model

for population growth . 405

maximum and minimum values , 199

mean life of an atom , 356

Mean Value Theorem , 208

Mean Value Theorem for integrals , 292

method of cylindrical shells , 377

method of exhaustion , 37

Midpoint Rule

error in using . 338,340

mixing problems , 406

model

See mathematical model.

moment

of a system of particles . 394,395

monotonic sequence , 418

Monotonic Sequence Theorem , 423

multiple integral

See double integral; triple integral.

multiplication of power series , 472

multiplier effect , 432

N

natural exponential function

power series for . 467,471

natural growth law , 168

natural logarithm function

limit of . 158

net area , 267

Net Change Theorem , 281

newton (unit of force) , 388

Newton's Law of Cooling , 172

Newton's Law of Gravitation , 400

Newton's method , 237

Newton's Second Law of Motion , 348

Newton, Sir Isaac , 286

newton-meter (unit of work) , 388

nondifferentiable function , 89

nonintegrable function , 335

normal density function , 220

normal line , 97

nth-degree Taylor polynomial , 465

numerical integration , 336

O

odd function , 221

one-sided limits , 29,30

one-to-one function , 149

optimization problems

strategy for solving . 227

order of a differential equation , 402

Oresme, Nicole , 429

orthogonal curves , 126

orthogonal trajectories , 126

P

p-series , 437

Pappus of Alexandria , 398

index

- parallelepiped , 366
 - paraxial rays , 135
 - partial fractions , 323
 - partial fractions, sum of , 324
 - partial integration , 307,308
 - partial sum of a series , 426
 - partition
 - regular . 266
 - parts, integration by , 307,308
 - pascal (unit of pressure) , 392
 - percentage error , 137
 - periodic function , 221
 - piecewise defined function , 5
 - point of inflection , 216
 - point-slope equation of a line , 74
 - Poiseuille's Law , 137
 - polynomial function
 - continuity of . 50
 - population growth , 405
 - position function , 76,90
 - pound (unit of force) , 388
 - power , 283
 - power function(s)
 - derivative of . 95
 - Power Law of limits , 36
 - Power Rule , 95,97,116
 - power series
 - representation of a function . 457
 - pressure exerted by a liquid , 391
 - prime notation , 77,98
 - Product Law of limits , 36
 - Product Rule , 106
 - profit function , 233
- ## Q
- quadratic function , 13
 - Quotient Law of limits , 36
 - Quotient Rule , 108
- ## R
- radian measure , 15
 - radioactive decay , 170
 - radiocarbon dating , 175
 - radius of convergence , 454,455
 - ramp function , 23
 - range of a function , 2
 - rate of change , 281
 - rate of cooling , 172
 - rate of decay , 170
 - rate of growth , 282
 - rate of reaction , 281
 - rates, related , 127
 - Ratio Test , 448
 - rational function
 - integration by partial fractions . 323
 - reciprocal function , 15
 - Reciprocal Rule , 113
 - rectilinear motion , 245
 - reduction formula , 311
 - reflection transformation a function , 17
 - region
 - between two curves . 361
 - regular partition , 266
 - related rates
 - strategy for solving . 129
 - relative error , 136
 - relative growth rate , 169
 - relative maximum and minimum , 200
 - remainder estimate for an alternating series, 445
 - remainder of the Taylor series , 466
 - removable discontinuity , 47
 - representation of a function:
 - as a power series . 457
 - revenue function , 232
 - revolution, solid of , 371
 - Riemann sum(s) , 264
 - Riemann, Georg Bernhard , 264,265
 - right cylinder , 366
 - right-hand limit , 30
 - Roberval, Gilles de , 278
 - Rolle's Theorem , 206
 - Rolle, Michel , 206
 - root function
 - continuity of . 50
 - Root Law of Limits , 36
 - Root Test , 450
 - rubber membrane, vibration of , 453
- ## S
- sample point , 258,264
 - scatter plot , 3
 - secant line , 74,75
 - second derivative , 90
 - Second Derivative Test , 217
 - separable differential equation , 402
 - sequence
 - term of . 415
 - series
 - trigonometric . 452
 - serpentine , 112
 - shell method , 377
 - shifts of functions , 17
 - Sierpinski carpet , 434
 - sigma notation , 258
 - simple harmonic motion , 120
 - Simpson's Rule
 - error bounds for . 344
 - Simpson, Thomas , 342,343
 - sine function
 - power series for . 468,471
 - sine integral function , 294
 - slant asymptote , 226
 - slope field , 407
 - slope of a curve , 74
 - slope-intercept equation of a line , 11
 - smooth curve , 382
 - solid of revolution , 371
 - solid, volume of , 371,372
 - solution curve , 407
 - solution of a differential equation , 402
 - speed , 79
 - spring constant , 390
 - Squeeze Theorem
 - for sequences . 419
 - stellar stereography , 356
 - step function , 6
 - strategy for optimization problems , 227
 - strategy for related rates problems , 129
 - stretching transformations of a function , 17
 - Substitution Rule
 - for definite integrals . 298
 - substitution, trigonometric , 317,318
 - sum
 - telescoping . 429
 - Sum Law of limits , 36
 - Sum Rule , 98
 - surface
 - quadric .
 - surface
 - quadric
 - (see quadric surface),
 - symmetric functions, integrals of , 299
 - symmetry , 299
 - symmetry principle , 395
 - system of particles, moment of , 394,395

index

T

table of integrals , 331
tabular function , 4
tangent function
 inverse . 178
tangent line
 vertical . 89
tangent line approximation , 133
Tangent Problem , 73
Taylor polynomial , 465,476
Taylor series , 463,464
Taylor's Formula , 466
Taylor, Brook , 464
telescoping sum , 429
term of a sequence , 415
term of a series , 425
term-by-term differentiation and integration ,
459
terminal velocity , 410
test for convergence/divergence of series
 Test for Divergence . 430
third derivative , 91
Torricelli's Law , 82,105
torus , 376,398
total fertility rate , 139
transformation
 of a function . 16
translation of a function , 17
Trapezoidal Rule
 error in . 338,340
trigonometric functions
 integrals of . 279
trigonometric integrals , 313
trigonometric substitutions in integration ,
317,318
Tschirnhausen cubic , 126

V

value of a function , 2
variable(s), change of , 295
variables, dependent and independent , 2
vector-valued function
 See vector function.
velocity
 instantaneous . 76
Velocity Problem , 76
Verhulst, Pierre-François , 405
vertical asymptote , 57

Vertical Line Test , 4
vertical tangent line , 89
vertical translation of a graph , 17
visual representations of a function , 2
volume
 of a solid of revolution . 371,372

W

washer method , 370,372
weight (force) , 389
weight density , 392
witch of Maria Agnesi , 112
work
 units for . 388,389
world population growth , 405