Homework 6
due Thurs., Mar. 19

(1) the problem you omitted on the midterm.

(2) #2 page 151

(3) This is a variant of problems 4 and 5 on page 151:
(3a) Plot the solution u in (3.8) for $0 \le t \le T$ with parameters $a = 1$, $b = 2$, $c = 37$, $T = 4$ and $f_0 = 1$. (3b) Show that

$$\hat{u}(\lambda) = \frac{1}{\sqrt{2\pi}} \left(\frac{f_0}{\omega a}\right) \left(\frac{1}{2i}\right) \left[\frac{1}{\mu + i(\lambda - \omega)} - \frac{1}{\mu + i(\lambda + \omega)}\right].$$

Now take 256 samples of u on the interval $[0, T]$ and compute $\hat{u}_k$ via the FFT. In matlab, subscripts start at 1, so the sampled function is

$$u_j = u(x_{j-1}), \qquad 1 \le j \le N, \quad x_j = \frac{Tj}{N}.$$

Be careful: we include the left endpoint of the interval but not the right when we sample a function. In Matlab, the assignment $v = \text{fft}(u)$ gives the vector with components

$$v_k = \sum_{j=1}^{N} u_j e^{-2\pi i (j-1)(k-1)/N}, \qquad 1 \le k \le N.$$

Note that the indices j and k are both "off by one" from their natural definitions. Next define the vector

$$\lambda_k = \left\{ \begin{array}{ll} \frac{2\pi}{T}(k-1), & 1 \le k \le N/2+1 \\ \frac{2\pi}{T}(k-1-N), & N/2+2 \le k \le N \end{array} \right\},$$

which maps the indices k to their natural place in Fourier space. (Read section 3.1.4 noting that here we write T instead of $b - a$. The T appearing in that section should really be called $\Delta T = T/N$). (3c) Plot $|\hat{u}(\lambda)|$ as a continuous curve over the interval $[-\Omega, \Omega]$ (using a lot of interpolation points, say 5000), where $\Omega = \frac{\pi N}{2T}$, and on top of that curve, plot the discrete points $\left(\lambda_k, \frac{\Delta T}{\sqrt{2\pi}}|v_k|\right)$ as markers. Make another plot zooming in on the "interesting" part of the graph (say for $\lambda \in [-20, 20]$ or so). (3d) Repeat (3a) and (3c) with $T = 16$ and $N = 1024$. The point of section 3.1.4 and 3.1.5 is that the markers lie close to the curve $|\hat{u}(\lambda)|$ if T and N are reasonably large. (3e) print out your code that made one of the plots. (This problem doesn't have to be done in Matlab – any programming language is fine.)

In case you weren't in class on March 12, you may "redo" the questions on the midterm that you missed to get half the points you missed. Each redo question is graded right/wrong with no partial credit. If you only missed part (b) of a question, for example, you only have to re-submit that part. The resubmissions are also due Thursday, March 19.