P. Vojta

Math 1B: Review Problems from Math 1A

Fall 2009

1. Compute the following limits:

   (a) $\displaystyle\lim_{x\to-2}\frac{x^3+8}{x+2}$ (d) $\displaystyle\lim_{x\to\infty} x\sin\frac{1}{x}$ (g) $\displaystyle\lim_{x\to0} e^{(\sin 3x)/2x}$

   (b) $\displaystyle\lim_{x\to-\infty}\frac{-6x^5+4x}{x^2+x+5}$ (e) $\displaystyle\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{1+\sin x}$ (h) $\displaystyle\lim_{h\to0}(1-2h)^{1/h}$

   (c) $\displaystyle\lim_{x\to\infty}\left(\sqrt{4x^2+5x}-\sqrt{4x^2+x}\right)$ (f) $\displaystyle\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{1-\sin x}$ (i) $\displaystyle\lim_{h\to0}\frac{\ln(1+h)}{h}$

2. State the δ-ϵ definition for the following conditions:
   (a) $\lim_{x\to a} f(x)=L$ (b) $\lim_{x\to a^-} f(x)=-\infty$

3. Let f be a function defined on $(-1,0)\cup(0,1)$ such that $\displaystyle\lim_{x\to0}\frac{f(x)}{x}=2$. Find $\displaystyle\lim_{x\to0}\frac{\sin x}{f(x)}$.

4. Compute the derivatives of the following functions from first principles (*i.e.*, using the definition of the derivative via limits): (a) $\sqrt{1+x}$ (b) $\frac{1}{3-x}$ (c) $\sqrt[3]{x}$

5. Differentiate the following functions: (a) $\log_3 x$ (b) $\tan\left(\frac{1}{1-x}\right)$ (c) $\ln(\ln x)$ (d) $x\sin^{-1}x$
   (e) $\sin(\sin^{-1}x)$ (f) $e^{\ln(x-1)}$ (g) 3^{x^2} (h) $x^{\sin x}$ (i) $\int_x^{1-x^2}\cos\sqrt{t}\,dt$

6. Find $\dfrac{dy}{dx}$ at $(1,1)$ if $y^3+3y=x^3+x+2$.

7. Use logarithmic differentiation (to its fullest!) to differentiate $\dfrac{x^2(x+1)^3}{\sqrt{2x+1}}$.

8. Find the point on the line $y=2x-3$ that is closest to the origin.

9. Use differentials to estimate: (a) $\frac{1}{.99}$ (b) $\sec\left(\frac{\pi}{6}+.02\right)$
   In part (b), is your estimate high or low? Use calculus.

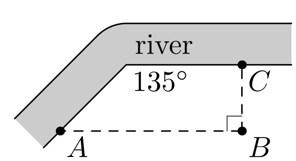
10. A circular disk is measured to be 10 inches in diameter, up to an error of ±0.1 inches. How accurately do we know its area?

11. A curve passes through the point $(0,5)$ and has the property that the slope of the curve at every point P is twice the y-coordinate of P. What is the equation of the curve?

12. We want to use Newton's method to find a solution of the equation $x^3+2x^2=17$.
    (a) Give the formula for going from one approximation x_n to the next one x_{n+1}.
    (b) If the initial approximation is 2, then find the next one.

13. Find $\sin^{-1}(\sin 79.3\pi)$.

14. A river has a 45° turn, as indicated in the picture. A rancher wants to construct a corral bounded on two sides by the river and on two sides by one mile of fence ABC as shown. Find the dimensions of the corral of largest area.



15. (a) Does Rolle's theorem apply to the function $f(x) = x + \sqrt{1-x^2}$ on the interval $[-1,1]$? Whether it applies or not, what would be the conclusion? Check directly whether this conclusion holds.
(b) Ditto for the Mean Value Theorem.

16. What is the minimum slope of $y = x^3 - 9x^2 + 15x$?

17. The Silicon Valley hot-spot, "High-Tech Ice Cream dot com" has an ice cream cone made in the shape of an inverted pyramid. Of all such shapes with fixed volume V, find the shape with minimal surface area (no top).

18. Plot the graph of $y = \frac{\sqrt{x}}{1+x}$ $(x \geq 0)$ and find the maximum value of y. Show all features of the graph (concavity, etc.).

19. Plot the graph of $y = \sqrt{x^2 + 2x}$ for $x \geq 0$ and prove that $y = x + 1$ is a slant asymptote.

20. Prove the inequality $\ln\left(1 + \dfrac{3}{x}\right) \geq \dfrac{3}{x+3}$, $x > 0$.

21. Does the function $f(x) = x^3 - 2x^2 + 2x - 1$ have an inverse? Carefully justify your answer. [Hint: think about what its graph looks like.]

22. Let f be an invertible function with the property that $f'(x)^2 = f(x) + 3$ for all x in its domain. Let $g(x)$ be the inverse of $f(x)$. Find $g'(x)$.

23. Two vertices of a triangle are $(5,0)$ and $(3,0)$. The third vertex moves along the line $y = 2x$ with a velocity of $\sqrt{20}$ units per second. At what rate is the area changing?

24. A ball is thrown straight up, with a speed of 64 ft/s from the top of a 96 foot cliff. Where is the ball t seconds later? When does it reach its maximum height? How high above the ground (bottom of the cliff) does the ball rise? When does the ball hit the ground? [Recall that the acceleration due to gravity is 32 ft/sec^2.]

25. Find: (a) $\displaystyle\int \frac{\sin x}{1+\cos^2 x}dx$ (b) $\displaystyle\int \frac{\sin x}{\sqrt{1+\cos^2 x}}dx$ (c) $\displaystyle\int_0^{13} \frac{dx}{\sqrt[3]{(27-2x)^2}}$

26. Find $\displaystyle\lim_{x\to\infty} \int_x^{x^2} e^{-t^2}\, dt$

27. The region between the curves $y = x^2$ and $y = x^3$ is revolved about the y-axis. Find the volume of the solid of revolution.

28. Find the volume of a right circular cone of radius r and height h, first by the disk method, then by the shell method.

29. The area bounded between $x^2 = 4y$ and $x + 2y = 4$ is revolved about the line $x = 3$. Find the volume.

30. Find the volume of a solid triangular pyramid whose base is a right triangle of sides 3, 4, and 5. The altitude of the pyramid is above the vertex A of the right triangle and has length 2.