Math 256A. Problem Set #11

Due Thursday, 19 November

1. Let k be a field, *not necessarily algebraically closed.* We recall that, if X is a scheme over k, then $X(\bar{k}) = X(\operatorname{Spec} \bar{k})$ is the set of k-morphisms from $\operatorname{Spec} \bar{k}$ to X.
   (a). Let $X = \mathbb{A}^n_k$ for some $n \in \mathbb{N}$. Points $P \in X(\bar{k})$ correspond to k-morphisms $\phi\colon \operatorname{Spec} \bar{k} \to X$, and the coordinate functions x_i on X pull back to give $a_i = \phi^* x_i \in \bar{k}$, which are called the **coordinates** of P: $P = (a_1, \ldots, a_n)$. Show that the induced map $\mathbb{A}^n_k(\bar{k}) \to \bar{k}^n$ is a bijection.
   (b). Let X be an integral closed subscheme of $\mathbb{A}^n_k$. Show that composing with the given closed immersion induces an injection $X(\bar{k}) \hookrightarrow \mathbb{A}^n_k(\bar{k})$.
   (c). Let X and Y be integral closed subschemes of $\mathbb{A}^n_k$ and $\mathbb{A}^m_k$, respectively. Let $p_1, \ldots, p_m$ be polynomials in $k[x_1, \ldots, x_n]$. If $P \in \mathbb{A}^n_k(\bar{k})$ has coordinates $(a_1, \ldots, a_n)$, then we let $p_i(P)$ denote the value $p_i(a_1, \ldots, a_n)$. Show that if $(p_1(P), \ldots, p_m(P))$ lies in $Y(\bar{k})$ for all $P \in X(\bar{k})$, then there is a unique k-morphism $\phi\colon X \to Y$ such that $\phi(P) = (p_1(P), \ldots, p_m(P))$ for all $P \in X(\bar{k})$.

2. Exercise 2.17.

3. Exercise 3.11a,c,d. For part (d), assume that Z is noetherian.

4. Exercise 3.12.