ERRATA ET ADDENDA
DIOPHANTINE APPROXIMATIONS AND VALUE DISTRIBUTION THEORY (SLN 1239)

Paul Vojta

1 July 2003; last revised 11 August 2017

Page iii, line 5

Change "number number field" to "number field."

Page iii, line 6

Change "localized at a finite number of places" to "localized away from a finite number of places."

Page iii, line 7

Change "k-rational solutions of the system" to "k-rational solutions of the system, for a number field k."

Page iv, line -10

Change $\mathbb{Q}$ to $\overline{\mathbb{Q}}$.

Page 2, equation (1.1.4)

[NEW 5 March 2011] Change $\|x\|_v$ on the left-hand side to $\|x\|_w$.

Page 4, line -5

Change "self intersection" to "self-intersection."

Page 4, line -2

(Comment.) This is known as *Kodaira's lemma.*

Page 7, line -14

Change $|f_i/f_j|^2$ to $|f_i/f_j|$.

Page 8, line 11

Change $\lambda_D \circ f$ to $\lambda_D \circ \phi$ and f^*D to ϕ^*D.

Page 8, line -1

[**NEW 17 September 2006**] This formula is correct (an earlier version of this list incorrectly stated that the formula held only for $v \nmid \infty$).

Page 9, line -7

Change "a hyperplane" to "the hyperplane at infinity."

Page 9, line -6

Change $\frac{1}{2[k:\mathbb{Q}]}$ to $\frac{[k_v:\mathbb{R}]}{2[k:\mathbb{Q}]}$.

Page 14, line -5

[**NEW 11 August 2017**] Change [**L 7**, Theorem 8.1] to [**L 7**, Ch. 2, Theorem 8.1].

Page 17, line 6

[**NEW 11 April 2006**] We assume $n > 0$.

Page 17, line 7

[**NEW 11 April 2006**] The set S must contain all archimedean places.

Page 17, line -6

Change $\prod_{v \in M_k}$ to $\prod_{v \in S}$.

Page 20, lines 5 and 6

Insert factors $\frac{1}{[k:\mathbb{Q}]}$ on the right-hand sides.

Page 22, lines 1–4

ρ should be taken relative to k.

Page 25, line -6

Delete the words "and $n_i \le 0$ otherwise." (This condition is not used subsequently.)

Page 26, line 14

Change "max" to "lub" (since we are dealing with a partially ordered set).

Page 27, line -7

Change this line to read, "$P_0 = [1 : 0 : 0 : 0]$, $P_1 = [0 : 1 : 0 : 0]$, and $P_2 = [0 : 0 : 1 : 0]$."

Page 34, line 6

[**NEW 7 September 2007**] In the right-hand column, change $\operatorname{ord}_v f$ to $\operatorname{ord}_v b$.

Page 34, Definition of counting function in left-hand column

The correct definition is

$$N(a,r) = \sum_{|w|<r} \operatorname{ord}^+(f-a) \log \frac{r}{|w|} .$$

Page 40, lines 4 and 11

Change $\displaystyle\int_0^r \frac{dt}{t}$ to $\displaystyle\int_0^r \frac{dt}{t^{2n-1}}$.

Page 47, line -3

Missing "." after "Conjecture."

Page 54, line -5

Change $CalO(1)$ to $\mathscr{O}(1)$.

Page 57, line 18

Change "ramaining" to "remaining."

Page 58, line 5

(Comment.) The bound $(1 + \log_p(n)) \log p$ comes from Serre, *Corps Locaux,* Ch. III, Remark 1 following Prop. 13.

Page 62, line 16

[**NEW 5 March 2011**] Change "local on X" to "local on V."

Page 62, line -10

[**NEW 5 March 2011**] Change $\mathbf{C}[[x_i, \ldots, x_n]]$ to $\mathbf{C}[[x_1, \ldots, x_n]]$.

Page 63–64, Conjecture 5.2.6

(Comment.) Isolated points of Z can be absorbed into the $O(1)$ term, so we may assume that all irreducible components of Z have dimension > 0.

Page 64, (generally)

(Comment.) For all smooth projective curves C there exists a reduced effective divisor D on C such that equality holds (up to $O(1)$) in the General Conjecture for points P of bounded degree, of heights $\to \infty$.

Proof. This is known if $C \cong \mathbb{P}^1$, so we may assume that $g(C) > 0$. Pick a nontrivial finite morphism $f\colon C \to \mathbb{P}^1$ and let B be the branch locus of f. Then B contains at least two points, which we may assume to be 0 and ∞. It is known that for sufficiently large S there are infinitely many S-units in k with heights $\to \infty$. Then apply the contrapositive of the argument on page 63. □

Remark. Even in the split function field case, I cannot see how to get this with $D = 0$. Indeed, we may assume that the base is $\mathbb{P}^1$; then we want maps

$$\begin{array}{ccc} C' & \xrightarrow{\ p\ } & C \\ {\scriptstyle\pi}\big\downarrow & & \\ \mathbb{P}^1 & & \end{array}$$

such that π has large degree, p has small degree, and p is almost étale. But a naïve count of degrees of freedom belies this: If $h' = \deg p$, and C' has s points of ramification, then $g(C') \ge h'(g-1) + s/2$. Also, the degree of the base locus on $\mathbb{P}^1$ is $2g(C') + 2\deg\pi - 2$, so the set of curves with a $g^1_{\deg\pi}$ is at most $2g(C') + 2\deg\pi - 3$ dimensional, and the set of curves C' covering C is s-dimensional, which should add up to $> 3g(C') - 3$, but it doesn't unless $h'/\deg\pi$ is small, approaching $2/(g-1)$.

See also Xiangjun Song's thesis.

Page 64, line 10

(Comment.) Note also that if (u, v) is a solution of $u+v = 1$ and is close to equality in the inequality $h(P) \le d(P) + O(1)$, then the binomial expansion of $(u+v)^n = 1$ gives a solution of $u_0 + \cdots + u_n = 1$ which is close to equality in the inequality $h(P) \le (\dim V)d(P) + O(1)$ for integral points on $\mathbb{P}^n \setminus \{(n+2)$ hyperplanes$\}$. (But, this is not a Zariski-dense example.) See Brownawell-Masser, *Math. Proc. Camb. Phil. Soc.* **100**, page 427.

Page 65, line 12

This follows from van der Waerden, § 5.9.

Page 65, line 14

This was actually proved for S of arbitrary (finite) size.

Page 68, lines 17–18

Change "such that the intersection of the supports of the principal divisors (f_i) is equal to D" to "such that the supports of the principal divisors (f_i) contain D for all i."

Page 70, equation (5.4.3.2)

Change $h(Q)$ to $h_{\pi^*\mathscr{O}(1)}(Q)$ (twice).

Page 70, line 10

Change $d(Q) \le N(R,Q) \le 2h(Q)$ to

$$\frac{1}{2}\log D_{k(Q)} = d(Q) \le N(R,Q) \le h_{\pi^*\mathscr{O}(1)}(Q)\ .$$

Page 71, Example 5.5.1

Inequality (5.5.0.1) implies a bound on n for which there is a *nontrivial* rational solution $[a:b:c]$; i.e., a solution whose coordinates are all nonzero.

Page 71, line -8

Change $N = Conductor(abc)$ to $N = \mathrm{Conductor}(abc)$.

Page 71, line -1

Change $[\sqrt[4]{a} : \sqrt[4]{b} : \sqrt[4]{c}]$ to $[\sqrt[n]{a} : \sqrt[n]{b} : \sqrt[n]{c}]$.

Page 80, Theorem 5.7.2

The divisor D doesn't need to have normal crossings, and the $(1,1)$-form ω is not holomorphic (which wouldn't make sense), just C^∞.

Page 80, line -4

Add at the end of the sentence: "which has the usual continuity properties."

Page 81, line 4

Change $(-1)^{p+q}$ to $(-1)^{p+q+1}$.

Page 83, line -4

[NEW 2 June 2017] Change 5.7.7 to (5.7.7).

Page 84, line -4

Change $\displaystyle\sum_{\substack{v\in S\\ v(abc)>0}}$ to $\displaystyle\sum_{\substack{v\notin S\\ v(abc)>0}}$.

Page 85, line -6

This line should read, "Then for all $m,n,x,y,z \in \mathbb{Z}$ with $(x,y)=1$ and satisfying ... "

Page 86, line -11

Change "For an elliptic curve" to "For a stable elliptic curve."

Page 88, line -2

Change "applies" to "implies."

Page 89, Chapter 6

Somewhere in Chapter 6 I should say something about Plücker formulas (Griffiths-Harris, pp. 269 and 270).

Page 90, Section 6.1

This section should be done using general convex symmetric bodies.

Page 91, line 3

Change [**Bo-V**] to [**B-V**].

Page 91, line 5

[**NEW 4 July 2019**] Add space after "however,".

Page 91, line 14

Change "volume 2^n" to "volume $2^n d(\Lambda)$."

Page 92, lines -13 and -12

[**NEW 4 July 2019**] Change $\mathscr{O}_{k,s}$ to $\mathscr{O}_{k,S}$ (twice).

Page 93, line 1

[**NEW 4 July 2019**] Change "S-integers" to "algebraic integers."

Page 94, line 14

[**NEW 4 July 2019**] Change (8) to (6.1.6).

Page 94, lines -12 and -11

Change [**Bo-V**] to [**B-V**].

Page 94, line -2

Change $N(\mathscr{D}_v^{-1})^{r/2}$ to $N(\mathscr{D}_v^{-1})^{n/2}$.

Page 95, line 5

Change [**Bo-V**] to [**B-V**].

Page 95, line 7

[**NEW 4 July 2019**] Change "$v \in S$" to "$v \notin S$."

Page 96, lines 4–7

These two paragraphs should not be italicized; they are not part of the statement of Theorem 6.1.11.

Page 97, equation (6.2.5)

[**NEW 5 March 2011**] Change $c_1\lambda_1\rho_i$ to $c_1\lambda_i\rho_i$.

Page 98, line 11

Change x to $\mathbf{x}$ (four times).

Page 103, line -1

[**NEW 24 April 2006**] In the upper-right entry of the big matrix on the left-hand side, B' should be B.

Page 104, line 17

Change $\epsilon \in 0$ to $\epsilon > 0$.

Page 104, line -11

Change $\|L_{v,i}(w_j)\|_v$ to $\|L_{v,i}(\mathbf{w}_j)\|_v$.

Page 104, line -11

change $0 \le 1 \le n$ to $0 \le i \le n$.

Page 104, lines -9 – -7

This paragraph should not be italicized; it is not part of the statement of Theorem 6.4.1.

Page 104, line -1

[**NEW 8 August 2020**] Change . (period) to "for all $1 \le j \le n$;".

Page 105, line 13

Change $\mathbf{x} \cdot b_i = 0$ to $\mathbf{x} \cdot \mathbf{b}_i = 0$.

Page 108, line 9

Change ρ_1 to ρ_i.

Page 111, line -7

Change (6.5.10) to (6.4.10).

Page 117, lines -6 – -1

Remark 6.5.12 looks at it wrong! We should actually look at it this way: $(x(t^2))' = x'(t^2)\cdot 2t$; $(x(t^2))'' = x''(t^2)\cdot 4t^2 + 2x'(t^2)$; etc.; the lower derivatives of $x(t^2)$ go away when you wedge, so the contribution of the ramification point is $(e-1)(n+1)(n+2)/2$. I don't see as much of a connection with the factor $\dim V$ anymore.

Page 120, line -9

Change "exists" to "exist."

Page 120, line -1

Change λ_1 to λ_i.

Page 123, Lemma 6.7.1

We may assume that $L_1, \dots, L_n$ are $x_1, \dots, x_n$, respectively. In order for this to make sense we really should be considering $\mathbf{x} \in \mathbb{C}[t]^n$; in that case the successive minima are all constants, so $\lambda_1 = \cdots = \lambda_n$. So the lemma is vacuous and should be omitted.

Page 124, [Ar 1]

This reference was in volume 5, not volume 35 (the original Russian article was volume 35, pp. 1269–1293).

Page 125, [G-K]

Change "146–220" to "145–220."

Page 128, [R]

This reference has a correction: *Mathematika* **2**, p. 168.

Page 128, [Sil 3]

This reference has appeared in *Invent. Math.* **81** (1985) 341–346; corrected *Invent. Math.* **84** (1986) 223.

Page 129, [Sil 4]

This reference has appeared in *Math. Ann.* **279** (1987) 193–216.

Page 129, [Sz 1]

Change 1971 to 1981.

Page 129, [Vo 2]

Cornell-Silverman is not in GTM.

Page 131, column 1, line -13

[NEW 2 June 2017] Change log^+ to $\log^+$ (wrong font).

Changes Since the Original Version of This List

(These changes also appear in the above list)

08/07/2005:

Page 111, line -7

Change (6.5.10) to (6.4.10).

09/17/2006:

Page 8, line -1

This formula was correct after all.

09/07/2007:

Page 34, line 6

In the right-hand column, change $\operatorname{ord}_v f$ to $\operatorname{ord}_v b$.

03/05/2011:

Page 2, equation (1.1.4)

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Page 97, equation (6.2.5)

Change $c_1\lambda_1\rho_i$ to $c_1\lambda_i\rho_i$.

06/02/2017:

Page 83, line -4

Change 5.7.7 to (5.7.7).

Page 131, column 1, line -13

Change log^+ to $\log^+$ (wrong font).

08/11/2017:

Page 14, line -5

Change [**L 7**, Theorem 8.1] to [**L 7**, Ch. 2, Theorem 8.1].

07/04/2019:

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Add space after "however,".

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