NAME ______________________________

STUDENT ID NUMBER ______________________________

TA's name or section number ______________________________

MATH 1B Final Exam Spring 2008

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There are 500 points altogether.

The first 15 questions are multiple choice, each worth 15 points. Choose the most correct answer to each question and mark the corresponding box in the grid ON THE BACK OF THIS PAGE. Mark only one box per question. No partial credit.

TA use only:

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Question	a	b	c	d	e
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Multiple choice questions:

1)Suppose the series $\sum_{n=0}^{\infty} a_n(-2)^n$ converges. We can conclude:

(a) $\sum_{n=1}^{\infty} a_n e^n$ converges.

(b) $\sum_{n=1}^{\infty} (-2)^n \cos a_n$ converges.

(c) $\sum_{n=1}^{\infty} n^3 a_n$ converges.

(d) $\sum_{n=1}^{\infty} |a_n|^{\frac{1}{n}}$ converges.

(e) $\sum_{n=1}^{\infty} |a_n|^{\frac{1}{n}}$ diverges.

2) The recurring decimal 0.212121212.... is the rational number

(a)3/11

(b)7/33

(c)21/212

(d)212/2121

(e)70/333

3) When trying to find a particular solution of

$$y'' + P(x)y' + Q(x)y = R(x)$$

one tries a function of the form $u_1y_1 + u_2y_2$ where y_1 and y_2 are solutions of the complementary (homogeneous) linear equation. Which of the following is most correct?

(a)The method can only work if P, Q and R are constants.

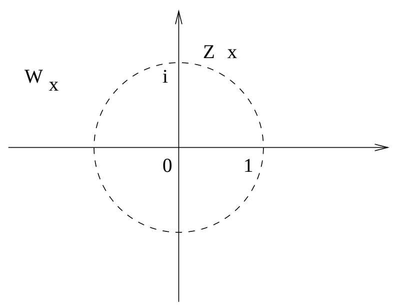
(b) The method can only work if P and Q are constants.

(c) One may impose the condition $u_1'y_1 + u_2'y_2 = 0$ since there are two unknowns and one can impose 2 equations

(d) The condition $u_1'y_1 + u_2'y_2 = 0$ follows from the original differential equation .

(e) The condition $u_1y_1' + u_2y_2'$ follows from the original differential equation.

(4) Which of the following is MOST CORRECT for the complex numbers Z and W marked with x's in the picture of the complex numbers below? (The dashed circle represents the unit circle - that is to say all complex numbers of modulus one.)
(a)$Z = W + 3i$
(b)$Z = W^2$
(c)$W = Z^2$
(d)$Z = 1/W$
(e)$Z = 2W$



5) The complex number e^{1+2i} is equal to

(a) $e^{\sqrt{5}}(\cos(\tan^{-1}(2)) + i\sin(\tan^{-1}(2)))$
(b) $e^{\sqrt{5}}(\cos(\tan^{-1}(2)) - i\sin(\tan^{-1}(2)))$
(c) $e(\cos 2 + i\sin 2)$
(d) $e^2(\cos 1 + i\sin 1)$
(e) a horse

6) Given three solutions y_1, y_2 and y_3 of the linear homogeneous differential equation $Py'' + Qy' + Ry = 0$, which of the following is true?
(a) For any constants c_1, c_2 and c_3, $c_1y_1 + c_2y_2 + c_3y_3$ is also a solution.
(b) y_1y_2 is always a solution.
(c) $y_1y_2y_3$ is always a solution.
(d) y_2y_3 is always a solution.
(e) You need to suppose $P(x) \neq 0$ for the differential equation to be linear.

7)Which of the following is true for any sequence $\{a_n\}$ with $\lim_{n\to\infty} a_n = \infty$?

(a) There is an $N > 0$ for which $a_n > 2$ for all $n \leq N$.
(b) There is an N for which $|a_n - 4| < 1$ for all $n \geq N$.
(c) $\lim_{n\to\infty}(a_n + a_{n+1}) = \infty$.
(d) For no value of n is a_n smaller than 300.
(e)For any $\epsilon > 0$ there is an N with $|a_n| < \epsilon$ for all $n \geq N$.

8)The integral $\int_1^\infty \frac{1}{x^2-4x+4}$ is

(a) divergent
(b) 1
(c) -1
(d) $\ln(|3|)$
(e) $\ln(3)$

9)The general solution to the differential equation $y''+4y'+5y=0$ is (where c_1, c_2, A and ϕ are arbitrary constants)

(a) $c_1 e^{2x} + c_2 e^{x}$

(b) $c_1 e^{2x} + c_2 e^{-x}$

(c) $e^{\sqrt{2}x}(c_1 \cos \frac{x}{\sqrt{5}} + c_2 \sin \frac{x}{\sqrt{5}})$

(d) $Ae^{-2x}\sin(x+\phi)$

(e) $Ae^{-x}\cos(4x+\phi)$

10) Pure water is poured at 2 litres per minute into a vat initially containing 100 litres of a salt solution with a concentration of 1gram per liter. The mixed solution is removed at one litre per minute. Which of the following initial value problems is correct for the the amount of salt S (in kg) in the vat?

(a) $2\frac{d^2S}{dt^2} + 100\frac{dS}{dt} + S = 1. \qquad S(0) = 0.1, S'(0) = 1$

(b)$2\frac{d^2S}{dt^2} + \frac{dS}{dt} + \frac{S}{100} = 1. \qquad S(0) = 0.1S'(0) = 1$

(c)$\frac{dS}{dt} = \frac{S}{100+t}. \qquad S(0) = 0.1$

(d)$\frac{dS}{dt} = -\frac{S}{100+t}. \qquad S(0) = 0.1$

(e)$\frac{dS}{dt} = \frac{S}{100} + t. \qquad S(0) = 0.1$

11) To integrate the function $\dfrac{x^3}{x^4-1}$ by partial fractions one should try to express it in the form

(a) $1 + \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+x+1}$

(b) $\dfrac{A}{x-1} - \dfrac{B}{x+1} + \dfrac{Cx+D}{x^2+1}$

(c) $\dfrac{A}{x^4} + \dfrac{B}{x^3} - \dfrac{C}{x^2} + \dfrac{D}{x}$

(d) $\dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+1}$

(e) $\dfrac{A}{x-1} - \dfrac{B}{(x-1)^2} + \dfrac{Cx+D}{x^2+1}$

12) Consider the differential equation $t^2\dfrac{dP}{dt} + P = 1$. Which of the following is the general solution for some arbitrary constant C?
(a) $P(t) = 1 + e^{1/t} + C$.
(b) $P(t) = 1 + e^{-1/t} + C$.
(c) $P(t) = 1 + C$.
(d) $P(t) = \int_C^t e^{\frac{x^3}{3}}\,dx$
(e) $P(t) = 1 + Ce^{1/t}$

13) Which of the following is correct concerning a linear homogeneous second order differential equation with constant coefficents?
(a) The boundary value problem always has a unique solution.
(b) The initial value problem always has a unique solution.
(c) The initial value problem may not have a solution but if one exists it is unique.
(d) The boundary value problem may not have a solution but if one exists it is unique.
(e) The boundary value problem always has a solution but there may be infinitely many different ones.

14) Which of the following integrals gives the area of the surface obtained by rotating the curve $y = \ln x$ for x between e and e^2 about the line $x = 9$?

(a)$2\pi \displaystyle\int_1^2 (9 - x)\sqrt{1 + (\ln x)^2}\, dx$

(b)$2\pi \displaystyle\int_e^{e^2} (e^x - 9)\sqrt{1 + e^{2x}}\, dx$

(c)$2\pi \displaystyle\int_e^{e^2} (9 - ln(y))\sqrt{1 + e^{2y}}\, dy$

(d)$2\pi \displaystyle\int_e^{e^2} (ln(y) - 9)\sqrt{1 + \frac{1}{y^2}}\, dy$

(e)$2\pi \displaystyle\int_1^2 (9 - e^y)\sqrt{1 + e^{2y}}\, dy$

15) $\displaystyle\lim_{x\to 0} \quad \frac{e^{x^2} - \cos x - \frac{3x^2}{2}}{ln(1 - x^4)}$ equals

(a) ∞
(b) $-\frac{11}{24}$
(c) 1
(d) $\frac{15}{32}$
(e) 0

The next six questions are *not* multiple choice. Show your reasoning and give your answers in the space provided.

1.(60 points)

Find two linearly independent solutions to the differential equation

$$y'' + xy = 0$$

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Don't worry about expressing your answer in terms of factorials. If you write down the first four terms of each series correctly and the pattern is clear you will get full credit.

2.(50 points)

How many terms of the series $\sum_{0}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ are necessary to compute that sum to within 0.01?

3.(40 points) Solve the initial value problem

$$y'' + 2y' + y = 0, \quad y(0) = 0, y'(0) = 1$$

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4. (40 points) Solve the differential equation $y' = \dfrac{\ln x}{xy + xy^3}$.

5)(40 points) Find a particular solution of the differential equation

$$y'' + y' + y = e^{2x} + 1$$

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6)(45 points-15 each) Evaluate the following integrals:

(i) $\int_0^2 \sqrt{4-x^2}dx$

(ii) $\int_1^\infty \frac{1}{x^e}dx$

(iii) $\int x(\ln x)^2 dx$