Qualifying Exam Syllabus

for Theo Johnson-Freyd

Proposed Exam date:
11am, 11 June 2009

Committee:

- Nicolai Reshetikhin (Advisor)
- Mark Haiman (Chair)
- Mina Aganagic
- Ori Ganor (Physics)

1 Major Topic: Lie Theory (Algebra)

References: [3, Chapters 0–3, 5, Appendices], [2]

1. Definition of Lie groups and Lie algebras. Relationship: tangent spaces, left-invariant vector fields, exponential map.
2. Solvable and nilpotent Lie algebras. Engel's theorem. Lie's theorem.
3. Simple and semisimple Lie algebras. Killing form. Cartan's criterion.
4. Levi decomposition. Lie's third theorem. Ado's theorem. Baker-Campbell-Hausdorff formula.
5. Universal enveloping algebra. Poincaré-Birkoff-Witt theorem.
6. Root systems. Weyl group. Cartan matrix. Dynkin diagrams. Classification of complex semisimple finite-dimensional Lie algebras.
7. Complete reducibility of finite-dimensional representations. Casimir element. Weyl Character Formula. Verma modules. Highest weight representations of semisimple Lie algebras.
8. Examples: A_n, $B_2 = C_2$, G_2

2 Major Topic: Quantum Groups (Algebra)

References: [1, Chapters 1–4, 8–9, 12–14], [4]

1. Formal power series. Poisson algebras, and quantization thereof.
2. Poisson manifolds. Symplectic leaves.
3. Lie bialgebras. Poisson Lie groups. Basic theory.
4. Classical Yang-Baxter equation. Triangular and quasitriangular Lie bialgebras.
5. Drinfeld's double of a Lie bialgebra. Standard Lie bialgebra structure on simple Lie algebras.
6. Hopf algebras. Drinfeld's double of a Hopf algebra. Quantized universal enveloping algebras.
7. Monoidal categories. Braidings. Ribbon categories. Application: invariants of framed tangles.

3 Minor Topic: Quantum Mechanics (Applied Mathematics)

Reference: [5, Chapters 1–5]

1. States, Hilbert space. Observables and measurements. Entaglement, uncertainty. EPR paradox and Bell's inequality.
2. Hamiltonian and time evolution. Schrödinger equation. Schrödinger versus Heisenberg picture. Simple Harmonic Oscillator.
3. Rotations, translations, and parity. Position and momentum. Angular momentum.
4. Semiclassical quantum systems. WKB states.

References

[1] P. Etingof and O. Schiffmann. *Lectures on quantum groups*. Second edition. Lectures in Mathematical Physics. International Press, Somerville, MA, 2002.

[2] M. Haiman. *Lie Groups*, UC Berkeley, Fall 2008. Lecture notes by T. Johnson-Freyd available at `http://math.berkeley.edu/~theojf/LieGroups.pdf`. Better notes are available at `http://math.berkeley.edu/~theojf/LieGroupsBook.pdf`.

[3] A.W. Knapp. *Lie Groups Beyond an Introduction*. Second edition. Progress in Mathematics, 140. Birkhäuser Boston, Inc., Boston, MA, 2002.

[4] N. Reshetikhin. *Quantum Groups*, UC Berkeley, Spring 2009. Lecture notes by T. Johnson-Freyd available at `http://math.berkeley.edu/~theojf/QuantumGroups.pdf`.

[5] J.J. Sakurai. *Modern Quantum Mechanics*. Revised edition. Addison-Wesley Publishing Company, Reading, MA, 1994.