

You must always justify your answers. This means: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. No justification = no points.

1. (5 pts) Evaluate the integral $\int \frac{x-1}{x^2+2x} dx$.

We decompose the integrand into partial fractions:

$$\begin{aligned} x^2 + 2x &= x(x+2) && \text{factoring} \\ \frac{x-1}{x^2+2x} &= \frac{A}{x} + \frac{B}{x+2} && \text{partial fraction ansatz} \\ x-1 &= A(x+2) + Bx && \text{cross-multiply} \\ 0-1 &= A(0+2) + B0 && \text{when } x=0 \\ -2-1 &= A(-2+2) + B(-2) && \text{when } x=-2 \\ \frac{x-1}{x^2+2x} &= \frac{-1/2}{x} + \frac{3/2}{x+2} && \text{solving for } A \text{ and } B \text{ and substituting} \end{aligned}$$

Thus, we can evaluate the integral, using the fact that $\int \frac{1}{x+a} dx = \ln|x+a| + C$:

$$\begin{aligned} \int \frac{x-1}{x^2+2x} dx &= \int \left(\frac{-1/2}{x} + \frac{3/2}{x+2} \right) dx \\ &= \boxed{-\frac{1}{2} \ln|x| + \frac{3}{2} \ln|x+2| + C} \\ &= \ln \sqrt{\left| \frac{(x+2)^3}{x} \right|} + C \end{aligned}$$

As with $\int dx/x$, the value of the constant C may change at any point of discontinuity; in this case, the value of C may change at $x = -2$ and at $x = 0$.

2. (5 pts) Evaluate the integral $\int \frac{x^2}{(4-x^2)^{3/2}} dx$.

We substitute $x = 2 \sin \theta$, restricting ourselves to $-\pi/2 < \theta < \pi/2$ (since $|x| \geq 2$ is outside the domain of the integrand). Then $dx = 2 \cos \theta d\theta$ and:

$$\begin{aligned} \int \frac{x^2}{(4-x^2)^{3/2}} dx &= \int \frac{(2 \sin \theta)^2}{(4-4 \sin^2 \theta)^{3/2}} 2 \cos \theta d\theta \\ &= \int \frac{8 \sin^2 \theta \cos \theta}{8 \cos^3 \theta} d\theta \\ &= \int \tan^2 \theta d\theta \\ &= \int (\sec^2 \theta - 1) d\theta \\ &= \tan \theta - \theta + C \\ &= \boxed{\frac{x}{\sqrt{4-x^2}} - \arcsin \frac{x}{2} + C} \end{aligned}$$