Math 1B Handout: Springs and circuits

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Springs

A spring with bobs back and fourth. If the spring has mass $m > 0$ and spring constant $k > 0$, and if it's moving through a fluid with viscosity $c > 0$, and driven by an external force $F(t)$, then it will move under a trajectory that solves a second-order differential equation:

$$my'' = F(t) - cy' - ky$$

This follows from Newton's lay: force equals mass times acceleration.

1. Let's say we have a spring with mass m, coefficient k, and viscosity c. It's rest position is $y(t) = 0$: this is the only state that doesn't move. Now let's say we hang it, so that the spring feels a constant external force of $f(t) = mg$. What is the new rest position? Find the general motion of the spring.

2. A spring with a one-gram bob and spring constant of two grams per second^2 is placed in a viscous medium, providing a damping force with coefficient equaling two grams per second. Moreover, let's say that the spring is driven by an external force of $\cos(t/\text{sec}) - 2\sin(t/\text{sec})$ dynes (gram-centimeters per second^2).

   Let's say the spring is released from two centimeters from its equilibrium position, with an initial velocity of one centimeter per second towards its equilibrium position. Then what is the spring's velocity when it first reaches its equilibrium position?

Circuits

A series circuit with a resistor (with resistance R), an inductor (with inductance L), a capacitor (with capacitance C), and an applied voltage $E(t)$ satisfies the differential equation

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E(t)$$

where Q is the charge in the capacitor.

1. Let's say that all three of L, C, and R are positive, with $R^2 < 4L/C$. If $E(t)$ is zero (except for some initial applied potential), then describe how Q changes with time.

2. Now let's say that, except for some initial applied potential, $E(t) = \sin(kt)$ for some k. What is the long-term behavior of the circuit?

3. What if $R = 0$ and $E(t) = 0$?

4. What if $R = 0$ and $E(t) = \sin(kt)$? For what k does $|Q|$ get larger and larger (on average)?