Math 32 Quiz

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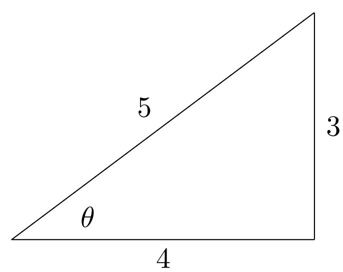
Name: ______________________________ Score: /10

You have thirty minutes to complete this quiz. You may not use calculators or notes, but the chalkboards are yours.

1. (3 pts) Let θ be defined by the following triangle:



Use the half-angle formulas to find $\tan(\theta/2)$. You do not need to simplify any radicals, but you do need to write your final answer without any trigonometric functions.

We recall the half-angle formulas:

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \frac{\sqrt{\frac{1}{2}\left(1-\cos\theta\right)}}{\sqrt{\frac{1}{2}\left(1+\cos\theta\right)}} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

The last line comes from multiplying the top and bottom of the fraction by $(1\pm\cos\theta)$ *and using the Pythagorean theorem.*

In any case, we have $\cos\theta = \frac{4}{5}$ *and* $\sin\theta = \frac{3}{5}$. *Thus*

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\frac{4}{5}}{1+\frac{4}{5}}} = \sqrt{\frac{1/5}{9/5}} = \boxed{\frac{1}{3}}$$

2. (3 pts) Use the addition formula to compute $\sin(75°) = \sin(45° + 30°)$.

$$\begin{aligned}
\sin(75°) &= \sin(45° + 30°) \\
&= \sin(45°)\cos(30°) + \cos(45°)\sin(30°) \\
&= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\
&= \boxed{\frac{\sqrt{6}+\sqrt{4}}{4}}
\end{aligned}$$

3. (4 pts) Use the sum-to-product formula to prove that the following equation is an identity:

$$\frac{\sin s + \sin t}{\cos s + \cos t} = \tan\left(\frac{s+t}{2}\right)$$

$$\boxed{\frac{\sin s + \sin t}{\cos s + \cos t} = \frac{2\sin\left(\frac{s+t}{2}\right)\cos\left(\frac{s2t}{2}\right)}{2\cos\left(\frac{s+t}{2}\right)\cos\left(\frac{s2t}{2}\right)} = \frac{\sin\left(\frac{s+t}{2}\right)}{\cos\left(\frac{s+t}{2}\right)} = \tan\left(\frac{s+t}{2}\right)}$$