# Math 32 Quiz

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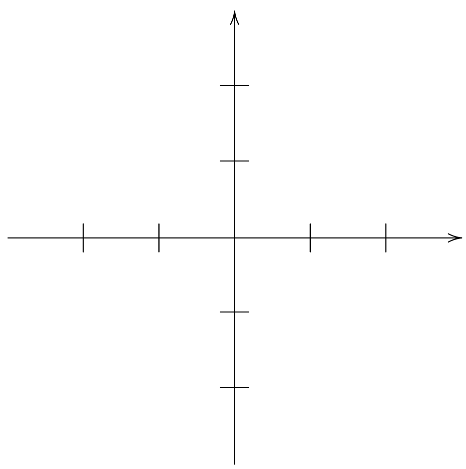
Name: ______________________________ Score: /10

*You have twenty minutes to complete this quiz. You may not use calculators or notes, but the chalkboards are yours.*

1. (3 pts) Sketch a graph of the function $F$:

$$F(x) = (x+2)^2 x(x-1)$$



*The important things to have in the graph are that the function goes to $+\infty$ in both directions, crosses the $x$-axis at $x = 0$ and $x = 1$, and touches but does not cross the $x$ axis at $x = -2$.*

2. (3 pts) Find the maximum value of the function $G$:

$$G(x) = \left(-x^2 + 4x - 1\right)^3$$

*The vertex of the parabola $y = -x^2 + 4x - 1$ is at $x = -4/-2 = 2$ and so $y = -2^2 + 4 \cdot 2 - 1 = 3$. Then the maximum value of $G(x)$ is $3^3 = \boxed{27}$.*

3. (4 pts) Set up an equation modeling the following word problem. Be very neat and clear: specify the meaning of each variable in your equation, and what's given and what you would need to solve for to answer the word problem. Use good handwriting — the idea here is that you could give this equation to a co-worker, who remembers how to solve equations (or program them on the computer) but not how to set them up. **You do not need to solve the equation.**

> A piece of wire 16 cm long is cut into two pieces. The first piece is formed into a rectangle in which the length is twice the width. The second piece is formed into a rectangle in which the length is three times the width. What should the lengths of the two pieces of wire be so that the total area of the two rectangles is minimized?

*We let $x$ denote the length of the first piece. Then $16 - x$ is the length of the second piece. The sides of the first rectangle are $a$ and $2a$, and the perimeter is $x = a + 2a + a + 2a = 6a$, so $a = x/6$; the area of the first rectangle is $2a^2 = 2(x/6)^2 = x^2/18$. The sides of the second rectangle are $b$ and $3b$ with $8b = 16 - x$, so $b = 2 - x/8$; the area is $3b^2 = 3(2 - x/8)^2$. Thus, the combined area of the two rectangles is*

$$A = \frac{x^2}{18} + 3\left(2 - \frac{x}{8}\right)^2$$

*and this is the function we want to minimize.*