

Math 1B Section 107 Quiz # 2

Thursday, 6 September 2007

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Name: _____

Evaluate the following integrals:

1. (3 pts) $\int \sin^3 x \cos^3 x dx$
- | | | |
|---|--|-------|
| $= \int \sin^3 x (1 - \sin^2 x) \cos x dx$ | $= \int \cos^2 x (1 - \cos^2 x) \sin x dx$ | .5 pt |
| $u = \sin x \quad du = \cos x dx$ | $w = \cos x \quad dw = -\sin x dx$ | .5 pt |
| $= \int (u^3 - u^5) du$ | $= \int (-w^3 + w^5) dw$ | .5 pt |
| $= \frac{u^4}{4} - \frac{u^6}{6} + B$ | $= -\frac{w^4}{4} + \frac{w^6}{6} + C$ | .5 pt |
| $= \frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + B$ | $= -\frac{\cos^4 x}{4} + \frac{\cos^6 x}{6} + C$ | 1 pt |
2. (3 pts) $\int \frac{dx}{x^3 \sqrt{x^2 - 1}}$
- | | | |
|---|--|-------|
| $x = \sec \theta \quad dx = \sec \theta \tan \theta d\theta$ | | .5 pt |
| $= \int \frac{\sec \theta \tan \theta d\theta}{\sec^3 \theta \sqrt{\sec^2 \theta - 1}} = \int \frac{\sec \theta \tan \theta d\theta}{\sec^3 \theta \tan \theta} = \int \frac{d\theta}{\sec^2 \theta}$ | | .5 pt |
| $= \int \cos^2 \theta d\theta = \int \frac{1}{2} (1 + \cos 2\theta) d\theta = \frac{\theta}{2} + \frac{1}{4} \sin 2\theta + C$ | | 1 pt |
| $= \frac{\arccos(1/x)}{2} + \frac{\sqrt{x^2 - 1}}{x^2} + C$ | | 1 pt |
3. (4 pts) $\int x \cos^2 x \sin x dx$
- | | | |
|--|--|------|
| $u = x \quad dv = \cos^2 x \sin x dx$ | | |
| $du = dx \quad v = \int \cos^2 x \sin x dx = -\int w^2 dw$ | | 2 pt |
| $= -w^3/3 = -(\cos^3 x)/3$ | | |
| $= -\frac{1}{3} x \cos^3 x + \frac{1}{3} \int \cos^3 x dx = -\frac{1}{3} x \cos^3 x + \frac{1}{3} \int (1 - \sin^2 x) \cos x dx$ | | 1 pt |
| $= -\frac{1}{3} x \cos^3 x + \frac{1}{3} \sin x - \frac{1}{9} \sin^3 x + C$ | | 1 pt |