

Homework 3, QFT, 276, Fall 2007

1. Let $f : M \rightarrow N$ be a morphism of super manifolds such that for some point $m \in M$, the differential $df_m : T_m M \rightarrow T_{f(m)} N$ is an isomorphism. Show that f is a local diffeomorphism around m .

Conclude that for any super Lie group G , the two maps between $\mathfrak{g} := \text{Lie}(G)$ and $T_e G$ given in class are inverses of each other. This should include an argument showing that ξ_v is left-invariant for $v \in T_e G$.

Furthermore, show that a homomorphism of super Lie groups $G \rightarrow G'$ induces a homomorphism of Lie algebras $\mathfrak{g} \rightarrow \mathfrak{g}'$ that is uniquely determined by the differential at $e \in G$.

2. Let \mathcal{E} be a vector bundle over a super manifold M , i.e. \mathcal{E} is a locally free sheaf of dimension $p|q$ over the sheaf \mathcal{O}_M . In this problem we'll try to define the *total space* of this bundle as morphism of super manifolds $p : E \rightarrow M$ which locally looks like $p_1 : U \times \mathbb{R}^{p|q} \rightarrow U$ (with fibrewise linear transition maps) and such that the sections of p give the sheaf \mathcal{E}^{ev} . There are two approaches:

- (a) Define E via its (sheaf of) functions $C^\infty(E)$ that are given as a completion of the sheaf of algebras $\text{Sym}_{\mathcal{O}_M}(\mathcal{E}^*)$. Here Sym_A is the free commutative super A -algebra functor, e.g. on even vector spaces it gives symmetric algebras and on odd vector spaces it gives exterior algebras.
- (b) Define an S -point of E to be a pair (f, s) where $f : S \rightarrow M$ is smooth and $s \in f^*(\mathcal{E})^{ev}$ is a 'section' of the pullback bundle. Then show that this leads to a representable functor $\text{SMAN} \rightarrow \text{SET}$ with the required properties.

PLEASE RETURN IN CLASS ON TUESDAY, OCT.9.