

Homework 2, QFT, 276, Fall 2007

1. Let $M^{m|n}$ be a super manifold and let $J \subset \mathcal{O}_M$ be the presheaf of ideals, generated by the odd elements in \mathcal{O}_M . Show that \mathcal{O}_M/J is a sheaf of algebras that defines a smooth structure M_{red} on $|M|$. Moreover, show that J/J^2 is a sheaf of modules over this sheaf of algebras. Conclude that there is a smooth vector bundle $E \rightarrow M_{red}$ of fibre dimension n and an isomorphism of super manifolds $M \cong \pi E$.

This is a theorem of Marjorie Batchelor at Cambridge.

2. Show that for any super manifold M , the pullback of coordinate functions induces a bijection between the morphisms $\text{SMAN}(M, \mathbb{R}^{p|q})$ and the set

$$\{(f_1, \dots, f_p, \varphi_1, \dots, \varphi_q) \mid f_i \in C^\infty(M)^{ev}, \varphi_j \in C^\infty(M)^{odd} \}$$

This is true even in the analytic setting but if you want, you can work in the smooth setting and use the fact that

$$\text{SMAN}(M, N) \cong \text{SALG}(C^\infty(N), C^\infty(M))$$

for any two super manifolds M, N .

PLEASE RETURN IN CLASS ON TUESDAY, SEPT. 25.