

Homework 1, QFT, 276, Fall 2007

1. Let M be a smooth manifold and ω_i two closed n -forms on M . Use Stokes' theorem to show that w_1 and w_2 are homologous if and only if they are concordant. (This requires you to first give precise definitions of both these notions.)
2. Let A be a commutative super algebra and V be a free A -module of super dimension $(m|n)$. This means that there is a basis with m even generators and n odd generators, i.e.

$$V \cong A^p \oplus \pi A^q.$$

Show that a morphism $f : V \rightarrow V$ in the category of A -modules is invertible if and only if the reduced map $f_{red} : V_{red} \rightarrow V_{red}$ is invertible. Here A_{red} is defined as the quotient of A by the ideal of nilpotent elements and one gets an A_{red} module $V_{red} := V \otimes_A A_{red}$.

For invertible morphisms, show that the formula for the *Berezinian* $\text{Ber}(f) \in A^{ev}$ given in class satisfies

- Ber is natural with respect to algebra homomorphism $A \rightarrow B$.
- Ber is multiplicative, i.e. $\text{Ber}(f \circ g) = \text{Ber}(f) \cdot \text{Ber}(g)$.
- $\text{Ber}(\text{id} + e \cdot f) = 1 + e \cdot \text{str}(f) \in A[e]$, where str is the super trace, and e is an even element with $e^2 = 0$.

Conversely, show that these properties completely determine Ber .

PLEASE RETURN IN CLASS ON TUESDAY, SEPT. 11.