

**DERIVED ALGEBRAIC GEOMETRY SEMINAR
BERKELEY, FALL 2006**

PART 1: Background

- Sept.14:** Homotopy theory of simplicial sets: Matthias Görner
- Definition of simplicial sets; simplicial homotopy groups; comparison to topological spaces; Dold-Kan correspondence.
 - Ref: May or Goerss-Jardine, Chap 1.
- Sept.21:** Infinity categories: Dave Spivak, **in 71 Evans from today on**
- Definition of infinity category; small categories are infinity categories, spaces are infinity categories, topological and simplicial categories are infinity categories; mapping spaces, homotopy categories of infinity categories; limits and colimits in infinity categories; Yoneda's Lemma; n-categories.
 - Ref: Lurie HTT Chap 1.
- Sept.28:** Derived categories and stable infinity categories: A.J. Tolland
- Definition of derived category, examples; definition of triangulated category, examples; t-structures and hearts, Definition of stable inf-category; the homotopy category of a stable inf-category is triangulated; exact functors, t-structures and localization; the inf-category of spectra is stable; derived inf-categories are stable.
 - Ref: Gelfand-Manin, Neeman, Weibel, Lurie, DAG1.
- Oct.5:** Stacks as sheafs of infinity categories: Valentin Tonita
- Functor of points philosophy, stacks as solutions to moduli problems; Grothendieck topologies; definition of stack, DM stack; examples of stacks: principle G-bundles, quotient stacks, moduli stack of elliptic curves. Grothendieck topologies on infinity categories; sheafs as morphisms between infinity categories that take certain colimits to limits.
 - Ref: Moerdijk; Hopkins; Hollander, Lurie HTT Chap 6.

PART 2: Derived Algebraic Geometry

- Oct.12:** Simplicial commutative rings: Arturo Prat-Waldron
- Definition of simplicial commutative ring; cotangent complexes; properties of scr's, their modules and algebras; topologies on the infinity category of simplicial commutative rings.
 - Ref: Lurie Thesis Sec 3, Sec 4.2.
- Oct.19:** Derived schemes and derived stacks: Aaron McMillian and Dave Brown
- The spectrum of a simplicial commutative ring; definition of derived scheme; relationship of derived schemes to classical DM stacks; the infinity category of derived schemes.
 - Ref: Lurie Thesis Sec 4, Sec 5.1.
- Oct.26:** The derived representability theorem: Xinwen Zhu
- Statement of the Artin representability theorem; statement of the derived Artin representability theorem.
 - Ref: Lurie Thesis Sec 7.
- Nov.2:** Derived intersection theory: Joel Kamnitzer
- Classical intersection theory: Bezout's theorem, Serre's intersection formula; classical deformation theory; derived perspective on intersection theory; derived perspective on deformation theory.
 - Ref: Lurie Thesis Sec 1.

PART 3: Spectral (or topological) Algebraic Geometry

- Nov.9:** E_∞ ring spectra: Fei Han
- Definitions and properties of prespectra, omega-spectra, spectra, coordinate-free spectra, symmetric spectra, orthogonal spectra, S-modules; associative and commutative ring spectra (or S-algebras), simplicial commutative ring as connective commutative HZ-algebras.
 - Ref: EKMM.
- Nov.16:** Spectral schemes and spectral stacks: Chris Pries
- Motivation via equivariant cohomology; the algebraic spectrum of a commutative ring spectrum; definition of spectral (derived) scheme; definition of spectral stack.
 - Ref: Lurie, Ell Sec 2, 3.
- Nov.30:** Spectral group schemes and oriented spectral elliptic curves: Scott Carnahan
- Definition of spectral group scheme (aka commutative group over a spectral derived scheme); preorientations and orientations of spectral group schemes; the multiplicative group example; the moduli problem of spectral elliptic curves.
 - Ref: Lurie, Ell Sec 3, Sec 4.0.
- Dec.5:** The spectral representability theorem: Jacob Lurie
- The spectral representability theorem; the spectral moduli stack of spectral elliptic curves; sketch of comparison of this stack to tmf .
 - **This talk will be on Tuesday, 2:10 - 3:30 in 939 Evans.**
- Dec.7:** A cool application of the theory: Jacob Lurie

REFERENCES

- [1] EKMM, Modern foundations for stable homotopy theory
- [2] Gelfand-Manin, Methods of Homological Algebra
- [3] Hollander, A Homotopy Theory for Stacks
- [4] Hopkins, Complex oriented cohomology theories and the language of stacks
- [5] Lurie, DAG1: Stable Inf-Categories
- [6] Lurie, Thesis: DAG
- [7] Lurie, Survey of Elliptic Cohomology
- [8] Lurie, Higher Topos Theory
- [9] Moerdijk, Introduction to the Language of Stacks and Gerbes
- [10] Neeman, Triangulated Categories
- [11] Weibel, Introduction to Homological Algebra