QUALIFYING EXAM SYLLABUS

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Major Topic: Stochastic Analysis (Probability)

Martingales and Optional Times: Filtrations and optional times; random time-change; martingale property; optional stopping and sampling; maximum and upcrossing inequalities; martingale convergence, regularity, and closure; limits of conditional expectations; regularization of submartingales.

Gaussian Processes and Brownian Motion: Existence and properties of Brownian motion; strong Markov and reflection properties; law of the iterated logarithm; recurrence and transience; Skorokhod embedding; Donsker's invariance principle.

Brownian Motion and Hausdorff dimension: Hausdorff dimension of graph, image and zero set of Brownian motion; Kaufman's doubling theorem; polar sets and capacity; intersections of two independent Brownian motions.

Stochastic Integrals and Quadratic Variation: Continuous local martingales; quadratic variation; existence and properties of integral; Itô's formula; approximation and uniqueness; random time-change.

Continuous Martingales and Brownian Motion: Martingale characterization of Brownian motion; random time changes of martingales; iterated and multiple integrals; integral representation of martingales; change of measure and Girsanov's theorem; Cameron-Martin theorem; Wald's identity.

Local Time: Semimartingale local time; Tanaka's formula; maximum process and local time for Brownian motion; regularization of local times; occupation density.

Stochastic Differential Equations: Strong solutions and the Itô theory; weak solutions; martingale problem and strong Markov property.

Connections with Partial Differential Equations: Harmonic function and Dirichlet problem; One-dimensional heat equation; Feynman-Kac formula.

References: Olav Kallenberg, *Foundations of Modern Probability*; P. Mörters, Y. Peres: *Brownian Motion*; I. Karatzas & S.E. Shreve, *Brownian Motion and Stochastic Calculus.*

Major Topic: Partial Differential Equations (Classical Analysis)

Laplace's Equation: Fundamental solution; mean value formulas; properties of harmonic functions; Dirichlet's principle.

Heat Equation: Fundamental solution; initial-value and nonhomogeneous problems; mean-value formula; regularity; energy methods.

First-Order Equations: Characteristics; local existence theorem.

Hamilton-Jacobi Equations: Hamilton's ODE; Legendre Transform; Hopf-Lax formula.

Conservation Laws: Shocks; entropy condition; Lax-Oleinik formula; weak solutions; uniqueness.

Sobolev Spaces: Hölder Spaces; weak derivatives; definition of Sobolev spaces; approximation by smooth functions; extensions; traces; Sobolev inequalities: Gagliardo-Nirenberg-Sobolev and Morrey; compact embeddings.

Second-Order Elliptic Equations: Definitions of elliptic equations and weak solutions; existence theorems for weak solutions; regularity; maximum principles.

Calculus of Variations: First and second variation; Euler-Lagrange equation; existence of minimizers; regularity; constraints; critical points.

Reference: L.C. Evans, *Partial Differential Equations*

Minor Topic: Dynamical Systems (Classical Analysis)

Invariant Measures and Ergodic Theorems: Ergodicity; Mixing; von Neumann and Birkhoff Ergodic Theorem.
Perron-Frobenious Operator, Liouville Equation
Entropy: Topological and metric entropy; Shannon-McMillan-Breiman theorem; Katok theorem.
Lyapunov Exponents: Kingman's subadditive ergodic theorem; Oseledets theorem.
Ergodicity of Hyperbolic Systems
Reference: F. Rezakhanlou, *Lecture Notes*