

Worksheet - exponential and logarithmic functions

Problem 1. Simplify given expressions:

a) $\frac{\ln(2^x)}{\ln 2}$

b) $\log_2 e^{\ln(2^x)}$

Problem 2. Find the limits:

a) $\lim_{x \rightarrow -\infty} \frac{2^x}{e^{x-5}}$

b) $\lim_{x \rightarrow 0} \ln(x^2 + 1)$

c) $\lim_{x \rightarrow \infty} \frac{e^{2x} - e^{-3x}}{e^{2x} - e^{-4x}}$

Problem 3. Find the inverse functions of

a) $f(x) = 2^{x^5}$

b) $f(x) = e^{\sqrt[3]{\ln x + 1}}$

Problem 4. Find $(f^{-1})'(0)$ if $f(x) = e^{e^x} - e$.

Problem 5. Assume $f: \mathbb{R} \rightarrow \mathbb{R}$ is a one-to-one function. Show that the following functions are one-to-one:

a) $g(x) = (f(x))^3$

b) $h(x) = f(x^3)$

Is $k(x) = f(x^2)$ one-to-one?

Assume that f satisfies $f(1) = 2$ and $f'(1) = 3$. Find $(g^{-1})'(8)$.