

Continuous functions

At which points are these functions continuous? What can you say about discontinuities?

1. $f(x) = \frac{x^3-1}{x-1}$

2. $f(x) = \sin(\sin(\sin(x)))$

3. $f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

Show that there the equation $\cos x = x$ has a root in the interval $(0, 1)$.

Show that there is a real number x such that $e^x = x^2$.

Let $f: [0, 1] \rightarrow \mathbb{R}$ be a continuous function such that $0 \leq f(x) \leq 1$, for all real numbers x from the domain. Show that f has a fixed point, that is there exist a point x_0 such that $f(x_0) = x_0$.

Show that there is a point on the equator which has the same air temperature as its antipodal point (that is the point diametrically opposite).