**Practice for the final**

**Solutions of all problems must by accompanied by relevant explanations.**

**Problem 1.** Find the limit

a) $\lim_{x\to 0} \dfrac{(\sin x)^2}{\sin(x^2)}$

b) $\lim_{t\to 0} \dfrac{\cos(\sin t) - 1}{t^2}$

**Problem 2.** Show that there exists a solution of the equation $e^x = x^2$.

**Problem 3.** Let $f$ be a differentiable function such that $f(0) = 4.1$ and $|f'(x)| \leq 2$. Show that the equation $f(x) = 0$ has no roots in the interval $[-2, 2]$.

**Problem 4.** Find the absolute minimum and the absolute maximum of $g(t) = t^3 - 2t^2 - 4t$ on $[-3, 3]$.

**Problem 5.** Find the local minimums, maximums and inflection points of the followinf functions

a) $f(x) = 3x - (3 - x)^4$

b) $g(x) = (x^2 - 5x + 7)e^x$.

**Problem 6.** Let $A$ be the point with coordinates $(0, 1)$, $B$ some point on the circle $(x + 1)^2 + y^2 = 1$ and $C$ the point diametrically opposite to $B$. Find the maximum possible perimeter of the triangle with vertices $A$, $B$, $C$ as we move the point $B$ along the circle.

**Problem 7.** A particle is moving with velocity $v(t) = \dfrac{t^5 - \sqrt{t}}{t^2}$ meters per second. Find the position of the particle $s(t)$ and total distance traveled 1 second after it started moving if the particle starts moving at $t = \dfrac{1}{2}$ seconds at the position $s(\dfrac{1}{2}) = 2$.

**Problem 8.** Show that $3 \leq \displaystyle\int_{-1}^{2} \sqrt{x^4 + 1} dx \leq 3\sqrt{17}$

**Problem 9.** Differentiate the function $G(t) = \int_t^{t^2} \frac{\sin x}{x} dx$

**Problem 10.** Find the average of the function $h(u) = \frac{1}{\sqrt{1-u^2}} + (u-7)^8$ on $[-\frac{1}{\sqrt{2}}, \frac{1}{2}]$.

**Problem 11.** Evaluate the indefinite integrals

a) $\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}} dt.$

b) $\displaystyle\int \frac{\sin(2u)}{1 + \sin^4 u} du$

**Problem 12.** Evaluate the integrals

a) $\displaystyle\int_e^{e^2} \frac{\ln x + 2}{x \ln x} dx$

b) $\displaystyle\int_{-\frac{\pi}{2}}^{\pi} \frac{\sin(2x)}{1 + \cos^2 x} dx$

**Problem 13.** Find the area of the region enclosed by the curves $x = y^2 - 3$ and $y = \frac{1}{3}(x - 1)$.

**Problem 14.** Find the volume of the solid obtained by rotating the region enclosed by the curves $y = x^2$ and $x = y^2$ around the line $x = -1$.

**Problem 15.** Find the volume of the solid obtained by rotating the region enclosed by the curves $y = \sqrt{x-1}$, $y = 0$ and $x = 5$ around the line $y = 3$.