HW 13 Solutions

Math 55, Professor Strain

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1 §7.4

(2) **(a)** Path, simple, not a circuit, length 4.

(b) Path, not simple, circuit, length 4.

(c) Not a path: no edge from d to b.

(d) Not a path: no edge from b to d.

(4) Yes.

(14) Algorithmic proof: Start at a vertex u of odd degree. In particular, u has at least one neighbor. Pick a neighbor v and go to it. If the neighbor has odd degree, we are done. Otherwise, its degree is even and nonzero so v has a neighbor w. Consider two cases. If $w = u$, we are back where we started and have used up two edges incident on u. Fortunately, u has odd degree so there is still an edge to try. Otherwise, we can continue until we reach a vertex of odd degree. (We need the graph to be finite so this algorithm will terminate.)

Abstract proof: Let v be a vertex of odd degree, and let H be the connected component of G containing v. Consider H as a connected graph in its own right, and apply the handshaking theorem: it must have another vertex u of odd degree. Since H is connected, there is a path from v to u.

(30) The adjacency matrix is built of n square blocks down the diagonal, with zeroes everywhere outside the blocks. Each entry of each block may be either 0 or 1. The size of each block is the number of vertices in the corresponding connected component.

2 §7.5

(14) Since every vertex has even degree, an Euler path exists. One example is $i, a, c, d, e, f, g, d, h, e, c, b, a, h, g, c, i, h, b$.

(20) Since each vertex (crossing of two lines) has even degree and the endpoints have odd degree 1, an Euler path exists.

(32) By Exercise 23, an Euler path from a to e exists: one such is $a, d, e, d, b, a, e, c, e, b, c, b, e$.

(36) **(a)** Each vertex has degree $n - 1$, which is even iff n is odd. Hence K_n has an Euler circuit iff n is odd and greater than 1.

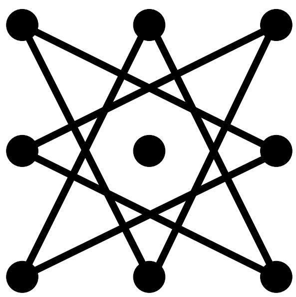
(b) Clearly C_n has an Euler circuit iff $n \geq 3$.

(c) No wheel has an Euler circuit since all the vertices around the rim have odd degree 3.

(d) Each vertex has degree n, which is even iff n is even. Hence Q_n has an Euler circuit iff n is even and greater than 1.

(44) No Hamilton circuit exists. If it did, it would have to go through the corners, which have degree 2, so it would have to include all the eight outer edges. Assume it started at a and went first to b (without loss of generality). If it goes next to vertex j in the interior part of the graph, then it can reach c only through edge dc and can never return. Similarly it must proceed around the outer edge until it reaches d. But then it cannot complete a circuit through all the interior vertices and still return to a, because edge ab was used first and edge da cannot be reached.

(52) The Hamilton path is d, c, a, b, e.



(64)

3 §7.6

(4) See the solution to Exercise 5: the length is 16.

(12) We could use Dijkstra's algorithm, but it's easy to see by inspection that the shortest path

- **(a)** from Boston to Los Angeles is the one through Chicago,
- **(b)** from New York to San Francisco is the one through Chicago,
- **(c)** from Dallas to San Francisco is the one through Denver or the one through Los Angeles,
- **(d)** from Denver to New York is the one through Dallas or the one through Chicago.

(24) Since there may be no shortest path if the weights are negative, Dijkstra's algorithm may not find it. However, even if there is a shortest path, negative weights may cause Dijkstra's algorithm to fail: consider the graph with vertices a, b and z where the weight of az is 2, the weight of ab is 3, and the weight of bz is -2. Dijkstra's algorithm will decide $L(z) = 2$ and stop, while the path abz is shorter with length 1.