HW Set 10 of Math 55, Spring 2001 (Prof. Strain)
6.1: 2,4,14,22,24,26,34
6.2: 2,6,8
6.3: 2,4,8,10,14
Solutions prepared by Nick Meyer
6.1:
2. a. $(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(5,5),(6,6)$
b, c. Ah, heck; graphs and tables are too tough to do. Look at Figure 2, page 376. It's not hard.

4. a. "Is taller than" is not reflexive (since I'm not taller than myself). It's not symmetric (I'm taller than Tom Cruise (who's 5'4"), but he's not taller than me). It is antisymmetric (since for different A and B, we never have A taller than B and B taller than A). It's also clearly transitive.
b. "Was born on the same day as" is trivially reflexive, symmetric, and transitive. It's not antisymmetric, since it's possible for different people to be born on the same day.
c. "Has the same name as" works exactly as in part b.
d. "Has a common grandparent as" is trivially reflexive and symmetric. It is not antisymmetric, since both my brother and I have the same grandparents, but we're not the same person. It is not transitive; I share a common grandparent with my cousin A, who shares a common grandparent with his cousin B (on the other side of the family tree), but I do not share a common grandparent with B.

14. a. $R^{-1} = \{(b,a)|a < b\} = \{(a,b)|a > b\}$
b. $\bar{R} = \{(a,b)|a \geq b\}$

22. Well, a relation on a set A is just a subset of $A \times A$, so the 16 relations on $\{0,1\}$ are simply the 16 subsets of $\{0,1\} \times \{0,1\} = \{(0,0),(0,1),(1,0),(1,1)\}$.

They are:
1. $\emptyset$
2. {(0,0)}
3. {(0,1)}
4. {(1,0)}
5. {(1,1)}
6. {(0,0), (0,1)}
7. {(0,0), (1,0)}
8. {(0,0), (1,1)}
9. {(0,1), (1,0)}
10. {(0,1), (1,1)}
11. {(1,0), (1,1)}
12. {(0,0), (0,1), (1,0)}
13. {(0,0), (0,1), (1,1)}
14. {(0,0), (1,0), (1,1)}
15. {(0,1), (1,0), (1,1)}
16. {(0,0), (0,1), (1,0), (1,1)}

24. We refer to the relevant subsets by the numbers in the previous exercise.
a. Reflexive: must contain $(0,0)$ and $(1,1)$; hence, subsets 8,13,14,16.
b. Irreflexive: must contain neither $(0,0)$ nor $(1,1)$; hence, subsets 1,3,4,9.
c. Symmetric: 1,2,5,8,9,12,15,16
d. Antisymmetric: 1,2,3,4,5,6,7,8,10,11,13,14
e. Asymmetric: 1,3,4

f. Transitive: 1,2,3,4,5,6,7,8,10,11,13,14,16

26. a. There are two relations on a set of one element (call it "a"): the empty set, and $\{(a,a)\}$. Both relations are trivially transitive.

b. There are 16 relations on a set of two elements, as we saw in Exercise 22; as we saw in 24f, thirteen of these sixteen are transitive.

c. There are $2^{3^2} = 512$ relations on a set of three elements; finding which are transitive is really terribly unpleasant. For the morbidly curious, 171 are transitive.

34. Let $A = \{1,2,3,4,5\}$. Then R^2 contains all pairs in $A \times A$ save $(2,3)$ and $(4,5)$, while R^3, R^4, R^5 contain all of $A \times A$.

6.2

2. There are 16 solutions to $abcd = 6$ in positive integers; the 4 permutations of $(6,1,1,1)$, and the 12 permutations of $(3,2,1,1)$.

6. Applying the projection $P_{2,3,5}$ yields columns 2,3,5.

8. Applying the projection $P_{1,2,4}$ yields columns 1,2,4 of Table 8; namely, those dealing with Airlines, Flight Numbers, and Destinations.

6.3

2. a. $(1,1),(1,3),(2,2),(3,1),(3,3)$

b. $(1,2),(2,2),(3,2)$

c. $(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(3,3)$

4. a. It's reflexive (and not irreflexive), since there are the main diagonal consists entirely of 1's. It's symmetric, since the matrix is symmetric about the main diagonal. It's not antisymmetric, since there is a pair of 1's symmetrically placed on either side of the main diagonal. It is transitive, which is easily seen by drawing the graph.

b. It's not reflexive, since there is a 0 on the main diagonal; it's not irreflexive, since there is a 1 on the main diagonal. It's not symmetric; it is, however, antisymmetric. It is easily seen to be transitive.

c. It's neither reflexive nor irreflexive. It is symmetric, but not antisymmetric. It's not transitive, since, for example, (2,1) and (1,2) are in the relation, but (2,2) isn't.

8. a. $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

b. $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

c. Taking the Boolean product of M_{R_1} and M_{R_2}, we obtain: $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

d. Taking the Boolean product of M_{R_1} and M_{R_1}, we obtain: $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

e. $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

10. Ack. More graphs; too hard to draw here. But it's really easy. Look at Example 8 (Figure 4).

14. $\{(a,a),(a,b),(b,a),(b,b),(c,a),(c,c),(c,d),(d,d)\}$.