Math 55, Section 1, Spring 2001
Professor Strain

Solutions for HW1

Section 1.1

6.

a) If you have the flu, you will miss the final examination.

b) To pass the course it is necessary and sufficient that you do not miss the final examination.

c) If you miss the final examination, you will not pass the course.

d) You have the flu, you miss the final examination, you pass the course, or any combination thereof.

e) You will not the pass the course if you have the flu, or you will not pass the course if you miss the final examination.

f) You will both have the flu and miss the final examination, or you will not miss the final examination and pass the course.

8

a) $p \wedge q$.

b) $p \wedge q \wedge r$.

c) $r \rightarrow p$.

d) $p \wedge \neg q \wedge r$.

e) $(p \wedge q) \rightarrow r$.

f) $r \leftrightarrow (p \vee q)$.

14.

a) "Are you a liar?" will evoke a negative answer in either case.

b) "If I were to ask you if you are a liar, would you respond with 'yes'?" (or something like that.

20.

a)

converse: If I stay home tonight, then it will snow.

contrapositive: If I do not stay home tonight, then it it not snowing.

b)

converse: It is a sunny summer day whenever I go to the beach.

contrapositive: If I do not go to the beach then it is not a sunny summer day.

c)

converse: When I sleep in until noon, it is because I stayed up late.
contrapositive: I get up in the morning only if I have not stayed up late.

26.

p	q	r	s	$p \to q$	$(p \to q) \to r$	$((p \to q) \to r) \to s$
T	T	T	T	T	T	T
T	T	T	F	T	T	F
T	T	F	T	T	F	T
T	T	F	F	T	F	T
T	F	T	T	F	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	T	T
F	F	T	F	T	T	F
F	F	F	T	T	F	T
F	F	F	F	T	F	T

Section 1.2

6.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

12.

p	q	$p \to q$	$\neg p$	$\neg p \wedge (p \to q)$	$\neg q$	$(\neg p \wedge (p \to q)) \to \neg q$
T	T	T	F	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

So this is not a tautology.

24. $p \wedge q \wedge \neg r$.

Section 1.3

10.

a) $\exists x \exists y \, Q(x, y)$.

b) $\forall x \forall y \, \neg Q(x, y)$.

c) $\exists x (Q(x, Jeopardy) \wedge Q(x, WheelofFortune))$.

d) $\forall y \exists x Q(x, y)$.

e) $\exists x \exists y (Q(x, Jeopardy) \wedge Q(y, Jeopardy))$.

12.

a) $\forall x F(x, Fred)$.

b) $\forall x F(Evelyn, x)$.

c) $\forall x \exists y P(x, y)$.

d) $\forall x \exists y \neg P(x, y)$.

e) $\forall y \exists x P(x, y)$.

f) $\neg \exists x (P(x, Fred) \wedge P(x, Jerry))$.

g) $\exists x \exists y (P(Nancy, x) \wedge P(Nancy, y) \wedge \forall z (P(Nancy, z) \rightarrow (z = x \vee z = y))$.

h) $\exists x (\forall y P(y, x) \wedge \forall z (P(z, x) \rightarrow z = x)$.

i) $\neg \exists x P(x, x)$.

j) $\exists x (\exists y P(x, y) \wedge x \neq y \wedge \forall z (P(x, z) \rightarrow z = y)$.

24.

a) $\forall y \forall x \neg P(x, y)$.

b) $\exists x \forall y \neg P(x, y)$.

c) $\forall x (\neg Q(y) \vee \exists R(x, y))$.

d) $\forall y (\forall x \neg R(x, y) \wedge \exists \neg S(x, y))$.

e) $\forall y (\exists x \forall z \neg T(x, y, z) \wedge \forall x \exists \neg U(x, y, z))$.

34.

a) $\forall x (P(x) \rightarrow \neg S(x))$.

b) $\forall x (R(x) \rightarrow S(x))$.

c) $\forall x (Q(x) \rightarrow P(x))$.

d) $\forall x (Q(x) \rightarrow \neg R(x))$.

e) Yes.

Section 1.4

6.

a) No

b) No

c) Yes

d) Yes

e) Yes

f) No

10. Let $A = \emptyset$ and $B = \{\emptyset\}$.

14. Yes. (Take the union of both sides.)

Section 1.5

20.

a) No. For example, take $A = \{1\}$, $B = \{2\}$, and $C = \{1, 2\}$.

b) No. Take any $A \neq B$ and let $C = \emptyset$.

38.

a) 0011100000

b) 1010010001

c) 0111001110

46. $n + 1$, unless the set is contained in itself, which is unlikely.