Thus $\gcd(p,q) = 1 \rightarrow \forall\, n \geq (p-1)(q-1)$

$\exists\, \underline{\underline{s, t \geq 0}}\ (n = sp + tq)$

E.g. $(2,3)$
$n \geq 2 \rightarrow$
$n = s \cdot 2 + t \cdot 3$
$=$

Proof WLOG assume $p < q$ and

$$(p-1)(q-1) \leq n \leq (p-1)q$$

(otherwise add q's as necessary). Then

$$m = n - p(p-1) \geq (q-p-1)(p-1)$$

So by structural induction $\gcd(p, q-p) = 1$ $\exists\, \sigma, \tau \geq 0$

$$m = \sigma(q-p) + \tau p$$

and

$$n = m + p(p-1) = \sigma q + (\tau - \sigma + p - 1)p.$$

Since

$$\sigma, \tau, p, q-p \geq 0 \quad \text{and} \quad m = \sigma(q-p) + \tau p$$

we have

$$\sigma(q-p) \leq m = n - p(p-1)$$

and

$$\sigma \leq \big(n - p(p-1)\big)/(q-p) \leq \frac{(p-1)q - p(p-1)}{q-p} = p-1$$

So $s = \sigma \geq 0$ and

$$t = \tau - \sigma + p - 1 \geq \tau \geq 0. \qquad \text{QED.}$$