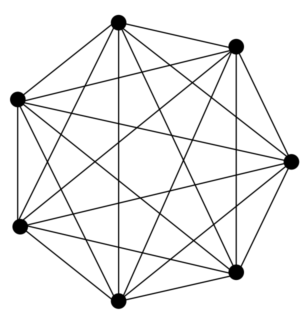
Math 55 - Homework 12 Solutions

8.1

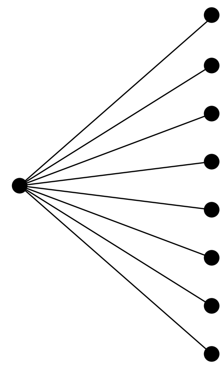
2. (a) simple graph (b) mutligraph (c) psuedograph
4. multigraph
6. multigraph
10. (4) remove (a,b), (b,d), (b,d) (5) remove (a,a), (a,b), (b,b), (b,d), (c,d), (d,d) (6) remove (a,c) (b,d)
12. crow, racoon, owl
16. influenced by deborah and yvonne, can influence brian

8.2

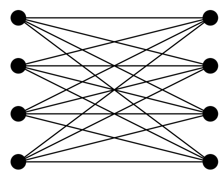
2. 5 vertices, 13 edges, degree a: 6 b: 6 c: 6 d: 5 e: 3
4. (1) 12 (2) 26 (3) 22
8. 4 vertices, 8 edges, (in,out) degree a: (2,2) b: (3,4) c: (2,1) d: (1,1)
18.



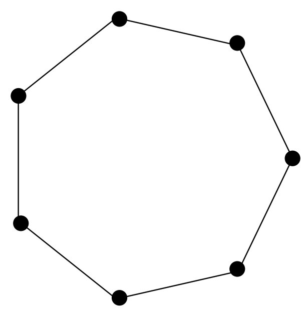
(a)



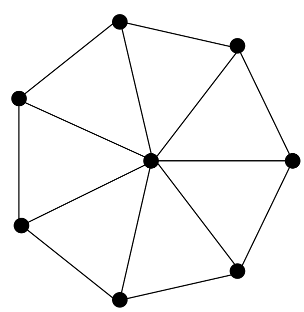
(b)



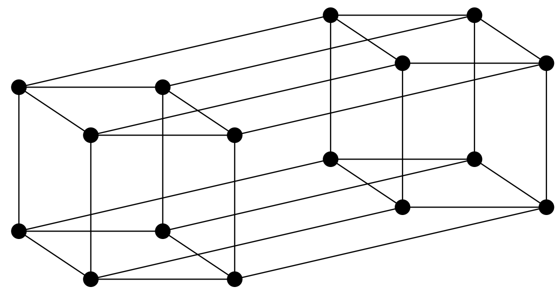
(c)



(d)

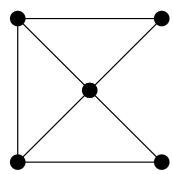


(e)

(f) 

22. yes

26. $4 + 3 + 3 + 2 + 2 = 14$, so there are 7 edges.



8.3

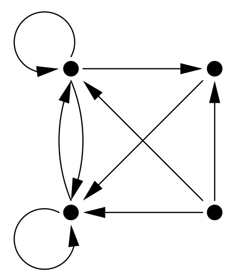
4.

Vertex	Adjacent vertices
a	b, d
b	a, c, d, e
c	b, c
d	a, e
e	c, e

8.

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

12.

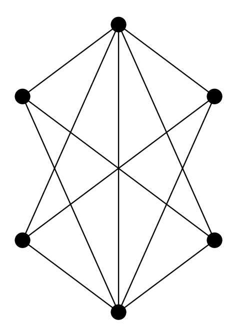


20.

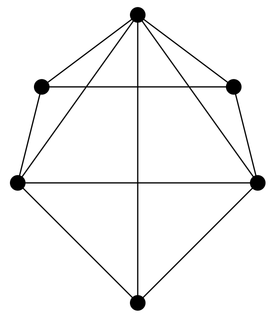
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

38. Yes, $\sigma = \{(u_1, v_1), (u_2, v_5), (u_3, v_2), (u_4, v_3), (u_5, v_4)\}$ is a graph isomorphism.

44. No, they're are not isomorphic. A simple proof is that the characteristic polynomials of their adjacency matrices are different. A more complicated (but elementary) proof goes like this: Suppose you take a vertex v from a graph, take its neighbor set $N(v)$, and then look at the subgraph generated by $N(v) \cup \{v\}$. If you do this to the first graph, you always get



If you do this to the second graph you always get



Since these are different, the two original graphs can't be isomorphic.

58. (a) yes (b) no