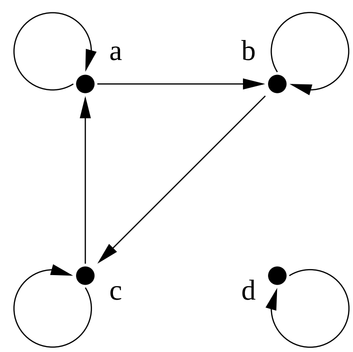
Math 55 - Homework 11 Solutions

7.4

2. $\mathbb{Z} \times \mathbb{Z}$.

4. Just add arrows from each to dot to itself, if none exists.

6.



10. same as 2.

12. The matrix representing $\{(a, a) \mid a \in A\}$ is I_n, and we know that $M_{R \cup S} = M_R \vee M_S$. Since the reflexive closure of R is $R' = R \cup \{(a, a) \mid a \in A\}$, we have $M_{R'} = M_R \vee I_n$.

16. (a) yes (b) no (c) yes (d) no (e) yes (f) no

26. (a) add $\{(a, a), (b, b), (c, c), (d, d), (e, b)\}$

    (b) add $\{(c, b), (b, b), (c, c), (e, e)\}$

    (c) $\{a, b, c, d, e\}^2$

    (d) $\{a, b, c, d, e\}^2$

7.5

2. (a) yes (b) yes (c) no (d) no (e) no

6. Let B be the set of R-equivalence classes of A, and let $f : A \to B$ be given by $f(a) = [a]_R$. This clearly does what we want.

10. Showing R is reflexive and symmetric is trivial from its definition. For transitivity, suppose $(a,b)R(c,d)$ and $(c,d)R(e,f)$. By definition

$$ad = bc \text{ and } cf = de$$

or, since all of these integers are positive,

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$$

Therefore, $af = be$ and $(a,b)R(e,f)$ as desired.

12. (a) Certainly $f^{(n)}(x) = f^{(n)}(x)$, $f^{(n)}(x) = g^{(n)}(x) \Rightarrow g^{(n)}(x) = f^{(n)}(x)$, and $f^{(n)}(x) = g^{(n)}(x) \wedge g^{(n)}(x) = h^{(n)}(x) \Rightarrow f^{(n)}(x) = h^{(n)}(x)$. So the relation is reflexive, symmetric, and transitive.

    (b) $\frac{d^4}{dx^4}(x^4) = 4!$, and for polynomials f, $f^{(4)} = 4!$ iff $f = x^4 + Ax^3 + Bx^2 + Cx + D$.

16. yes, it is.

18. (a) no (b) yes (c) yes

22. (a) Every class is of the form $\{f : \mathbb{Z} \to \mathbb{Z} \mid f(a) = C\}$ for some $C \in \mathbb{Z}$.

    (b) Every class is of the form

$$\{g : \mathbb{Z} \to \mathbb{Z} \mid \exists C \in \mathbb{Z}\ \forall x \in \mathbb{Z}\ g(x) = f(x) + C\}$$

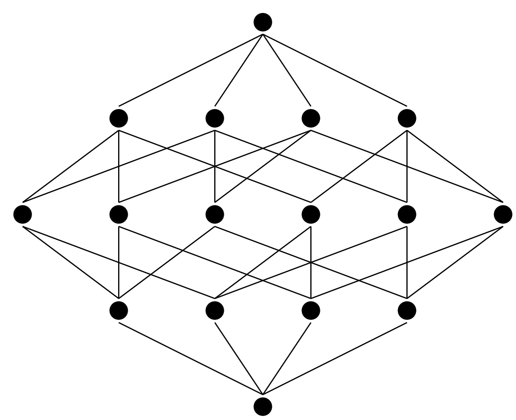
    for some $f : \mathbb{Z} \to \mathbb{Z}$.

26. (a) $\{n \in \mathbb{Z} \mid 2 \mid n\}$ (b) $\{n \in \mathbb{Z} \mid 3 \mid (n+1)\}$ (c) $\{n \in \mathbb{Z} \mid 6 \mid (n+2)\}$ d) $\{n \in \mathbb{Z} \mid 8|(n+4)\}$

7.6

2. (a) yes (b) yes (c) no

18.



26. (a) m, l (b) a, b, c (c) no (d) no (e) k, l, m (f) k (g) none (h) none

30. (a) $(\mathbb{N}, \leq)$ (b) $(\mathbb{N}, \geq)$ (c) $(\mathbb{Z}, \leq)$.

34. (a) If a and b are both greatest, then $a \leq b$ and $b \leq a$. By the antisymmetry of $\leq$, it must be that $a = b$.

    (b) Same argument: If a and b are both least, then $b \leq a$ and $a \leq b$.