Math 55 Solutions to Homework 2

Chapter 1

Section 6

6. A. Yes B. No C. Yes D. No E. No F. No

12. Let A={1} and B={1,{1}} . (Answers will vary).

16. Yes. Suppose $A \neq B$. Then there exists $x \in A$ such that $x \notin B$ or vice versa. Then $\{x\} \in P(A)$ but $\{x\} \notin P(B)$ (or vice versa). Hence $P(A) \neq P(B)$. By the contrapositive, if P(A) = P(B) then A=B.

Section 7

22. A. No, consider A={1}, B={2}, C={1,2,3}
B. No, consider A={1,3}, B={2,3}, C={3}

40. A. 0011100000
B. 1010010001
C. 0111001110

48. n+1

Section 8

10. A. Yes
B. No, f(a)=f(b)
C. No, f(a)=f(d)

12. A. Yes
B. No, f(-1)=f(1)
C. Yes
D. No, f(1)=f(2)

28. $f(g(x)) = (x+2)^2 +1 = x^2+4x+5$
$g(f(x)) = (x^2+1)+2 = x^2+3$

30. f(g(x)) = a (cx+d) + b = acx+ad+b g(f(x)) = c (ax+b)+d = acx+bc+d
So f(g(x)) = g(f(x)) when acx+ad+b = acx+bc+d which simplifies to ad+b = bc+d

34. A. {-1,1}
B. $\{x \mid -1<x<1, x\neq 0\}$

C. {x | x<-2 or x>2}

64. We wish to show f is injective $\Leftrightarrow$ f is surjective
($\Rightarrow$) f is injective. So $|F(A)| = |A|$. (Ideally, this fact should be proved by mathematical induction, but that isn't covered until 3.3). Since $|A| = |B|$ we get that $|F(A)| = |B|$. Since we also have that $F(A) \subseteq B$, and B is finite, we conclude $F(A) = B$ so f is surjective.
($\Leftarrow$) f is surjective. So $|F(A)| = |B|$. Since $|A| = |B|$ we get that $|F(A)| = |A|$. Since A is finite, this implies A is injective (again, ideally proved by induction).

Chapter 2

Section 1

2. Answers may vary slightly since some of the classifications can be arbitrary. We will abbreviate I input, O output, D definiteness, C correctness, F finiteness, E effectiveness, G generality.

A. Has: I, O, D, E, G	Lacks: C, F	Procedure never stops
B. Has: O, D, F, G	Lacks: I, C	No input for m, divides by 0 at end
C. Has: O, D, E, G	Lacks: I, C, F	No input for i, never stops
D. Has: I, O, C, F, E, G	Lacks: D	

10. Procedure PositivePower (x:real n: nonnegative integer)
```
PositivePower :=1
For i := 1 to n
PositivePower := PositivePower * x
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Procedure Power (x real n: integer)
If x=0 and n<1 then End {value can't be determined}
If n<1 then Power := 1 / PositivePower (x, -n)
If n>0 then Power := PositivePower (x, n)
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24, Procedure Injection? (A: set, n: cardinality of A, f: function)
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Injection? := True
For i :=2 to n
For j :=1 to i-1
If f(A(i))=f(A(j)) then Injection? := False
```

Section 2

8. A. n=4

B. n=5
C. n=1
D. n=-1

10. For $x>1$ we note $x^3<x^4$. Letting C=1, k=1 we get x^3 is $O(x^4)$.
Suppose x^4 is $O(x^3)$. Then $x^4 \leq Cx^3$ for $x>k$ for some positive constatnts C and k.
Dividing each side by x^3 we get $x \leq C$ for $x>k$ contradicting C being a constant.
Therefore x^4 is not $O(x^3)$.

18. We observe $1^k + 2^k + . . . + n^k \leq n^k + n^k + . . . + n^k = n(n^k) = n^{k+1}$. Letting C=1 we have that it is $O(n^{k+1})$.

60. As suggested by the hint we note that $x \leq$ ceiling(x) for all $x>1$.
So $1 / x \geq 1$ / ceiling(x) for all $x>1$. Hence Integral(1 / x) $\geq$ Integral (1 / ceiling(x)) over any finite interval above 1. Thus for any positive integer n, 1 + ln(n) $\geq$ Hn. From this we see that Hn $\leq$ 2(ln(n)) for any n>3. Therefore letting C=2 and k=3 we have that Hn is O(ln(n)).

Section 3

4. k - 1 multiplications are used. This is much more efficient than the alternative which requires 2^k - 1 multiplications.

6. The procedure counts the number of iterations of S := S AND (S-1) before S=0. To show the procedure works, it suffices to show that if S has n 1' s then S AND (S-1) has n-1 1' s. We note S and S-1 agree before the rightmost 1 of S. On this digit S-1 has 0 while S has 1 and after this digit S has only zeroes (because it was the rightmost 1).
So S and S AND (S-1) agree with eachother on every digit except the rightmost 1 of S which is a 0 in S AND (S-1). Hence if S has n 1' s then S AND (S-1) has n-1 1' s so the procedure works.
The procedure requires as many AND operations as there are 1' s in the string.

8. A. y= ((3(2)+1)(2))+1 = 15
B. n multiplications and n additions are used.