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Math 55 HW 1 Solutions

Chapter1:

1-8.

A. If you have the flu you will miss the final examination.

B. You pass the course if and only if you don't miss the final.

C. If you miss the final you won't pass the course

D. You have the flu or you will miss the final examination or you will pass the course

E. Either if you have the flu you won't pass the course or if you miss the final you won't pass the course (or both)

F. You have the flu and miss the final or you don't miss the final and pass the course.

10.

A. $r \wedge \neg q$

B. $p \wedge q \wedge r$

C. $\neg p \rightarrow \neg r$

D. $p \wedge \neg q \wedge r$

E. $(p \wedge q) \rightarrow r$

F. $(p \vee q) \leftrightarrow r$

22.

A. Converse: If I stay at home, then it will snow tonight.
Contrapositive: If I don't stay home, then it didn't snow tonight.
Inverse: If it doesn't snow tonight then I won't stay at home.

B. Converse: If I go to the beach, then it is a sunny summer day.
Contrapositive: If I don't go to the beach, then it wasn't a sunny summer day.
Inverse: If it isn't a sunny summer day, then I don't go to the beach.

C. Converse: If I needed to sleep until noon then I stayed up late.
Contrapositive: If I didn't need to sleep until noon then I didn't stay up late.
Inverse: If I didn't stay up late then I didn't need to sleep until noon.

24.

A,B.

p	$\neg p$	$p \rightarrow \neg p$	$p \leftrightarrow \neg p$
T	F	F	F
F	T	T	F

C.

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

D.

p	q	$p \vee q$	$p \wedge q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	F	T

E.

p	q	$\neg p$	$q \rightarrow \neg p$	$p \leftrightarrow q$	$(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

F.

p	q	$\neg q$	$p \leftrightarrow \neg q$	$p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	T	F	T
F	F	T	F	T	T

42. A. Either cannibal would answer no.
B. If I asked you if you were a liar, how would you reply? (The truth teller will say no, the liar yes). A trivial fact such as Is 1+1=2? would also work. Other answers possible.

2: 6.

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \vee \neg q$	$\neg(p \wedge q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

12. $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$	Statement
$\neg(\neg p \wedge (\neg p \vee q)) \vee \neg q$	Implication (twice)
$(\neg\neg p \vee (\neg\neg p \wedge \neg q)) \vee \neg q$	De Morgan (twice)
$(p \vee (p \wedge \neg q)) \vee \neg q$	Double Negation (twice)
$(p \vee \neg q) \vee (p \wedge \neg q)$	Associative / Commutative
$(p \vee \neg q \vee p) \wedge (p \vee \neg q \vee \neg q)$	Distributive
$p \vee \neg q$	Idempotence (thrice)

Hence the statement is not a tautology. This problem could also be done by truth tables.

34. $p \wedge q \wedge \neg r$

3:10. A. $\exists x(Cx \land Dx \land Fx)$

B. $\forall x(Cx \lor Dx \lor Fx)$

C. $\exists x(Cx \land Fx \land \neg Dx)$

D. $\neg\exists x(Cx \land Dx \land Fx)$

E. $\exists x(Cx) \land \exists x(Dx) \land \exists x(Fx)$

12. A. True B. True C. False D. True E. False F. True G. False

24. A. U.D. your school Vx: x has visited Uzbekistan $\exists x(Vx)$

U.D. people Sx: x is in your school Vx: x has visited Uzbekistan $\exists x (Sx \land Vx)$

U.D. people and nations Vxy: person x has visited nation y

Sx: person x is in your school u: Uzbekistan $\exists x(Sx \land Vxu)$

B. U.D. your class Cx: x has studied calculus

Px: x has studied C++ $\forall x(Cx \land Px)$

U.D. people Lx: x is in your class Cx, Px same as above $\forall x(Lx \to (Cx \land Px))$

U.D. people and subjects Sxy: person x studies subject y

Cx: person x is in your class c: Calculus p: C++ $\forall x(Cx \to (Sxc \land Sxp))$

C. U.D. your school Bx: x owns a bicycle

Mx: x owns a motorcycle $\forall x(\neg(Bx \land Mx))$

U.D. people Sx: x is in your school Bx, Mx same as above $\forall x(Sx \to \neg(Bx \land Mx))$

U.D. people and vehicles Sx: person x is in your school

Oxy: person x owns vehicle y b: bicycle m: motorcycle

$\forall x(Sx \to \neg(Oxb \land Oxm))$

D. U.D. your school Hx: x is happy $\exists x(\neg Hx)$

U.D. people Sx: x is in your school Hx: x is happy $\exists x(Sx \land \neg Hx)$

U.D. people and emotions Sx: person x is in your school

Fxy: person x has emotion y h: happiness $\exists x(Sx \land \neg Fxh)$

E. U.D. your school Bx: born in 20th century $\forall x(Bx)$

U.D. people Sx: x is in your school Bx: born in 20th century $\forall x(Sx \to Bx)$

U.D. people and centuries Sx: person x is in your school

Bxy: person x was born in century y t: twentieth $\forall x(Sx \to Bxt)$

38. A. U.D. positive rational numbers and people

Mx: x is the number of megabytes free on the hard drive

Lxy: x is less than y (both numbers) t: thirty

Ux: person x is a computer system user Wx: a warning message is sent to person x

$\exists x(Lxt \land Mx) \to \forall x(Ux \to Wx)$

B. U.D. directories in the file system, files (in the system), and system errors

Dx: x is a directory Fx: x is a file Tx: x is a detected system error

Ox: x can be opened Cx: x can be closed

$\exists x(Tx) \to \forall x((Dx \to \neg Ox) \land (Fx \to \neg Cx))$

C. U.D. systems and users

Sx: system x can't be backed up   Lx: user x is logged on   f: file system

$\exists x(Lx) \rightarrow \neg Sf$

D. U.D. positive rational numbers and services

Lxy: x is less than or equal to y (both numbers)   e: eight   f: fifty six

Dx: service x can be delivered   v: video on demand

Mx: x is the number of megabytes of memory available

Cx: x is the number of kilobits per second of connection speed

$\exists x \exists y(Mx \wedge Lex \wedge Cy \wedge Lfy) \rightarrow Dv$

58. A. $\forall x(Px \rightarrow \neg Sx)$

B. $\forall x(Rx \rightarrow Sx)$

C. $\forall x(Qx \rightarrow Px)$

D. $\forall x(Qx \rightarrow \neg Rx)$

E. Yes. (Let x be one of your poultry. Then it is a duck by C so it doesn't waltz by A so it isn't an officer by B).

4:6. A. Randy Goldberg is enrolled in CS 252.

B. Somebody is enrolled in Math 695.

C. Carol Sitea is enrolled in some course.

D. Someone is enrolled in both Math 222 and CS 252.

E. There are two different people such that the second person is enrolled in every class that the first person is enrolled in.

F. There are two different people who are both enrolled in the same classes.

10. f: Fred   e: Evelyn   j: Jerry   n: Nancy

A. $\forall x(Fxf)$

B. $\forall x(Fex)$

C. $\forall x \exists y(Fxy)$

D. $\neg \exists x \forall y(Fxy)$

E. $\forall x \exists y(Fyx)$

F. $\neg \exists x(Fxf \wedge Fxj)$

G. $\exists x \exists y(x \neq y \wedge Fnx \wedge Fny) \wedge \forall x \forall y \forall z((Fnx \wedge Fny \wedge Fnz) \rightarrow (x=y \vee x=z \vee y=z))$

H. $\exists x \forall y(Fyx) \wedge \forall x \forall y(\forall z(Fzx \wedge Fzy) \rightarrow x=y)$

I. $\forall x \neg(Fxx)$

J. $\exists x \exists y(Fxy \wedge \forall z(Fxz \rightarrow y=z))$

26. A. False   B. True   C. False   D. False   E. True

F. True (for any x, let y=0)   G. True (y=0)   H. False   I. False

5:4. r=It rains f=It is foggy s=The sailing race will be held
d=The lifesaving demonstration will go on t=The trophy is awarded

1. $\neg t$	Given
2. $s \to t$	Given
3. $\neg s$	Modus Tollens (1,2)
4 $(s \wedge d) \to s$	Simplification (T)
5. $\neg(s \wedge d)$	Modus Tollens (3,4)
6. $(\neg r \vee \neg f) \to (s \wedge d)$	Given
7. $\neg(\neg r \vee \neg f)$	Modus Tollens (5,6)
8. $r \wedge f$	De Morgan, Double Negation
9. r	Simplification (8)

12. A. Correct (Modus Tollens)
B. Incorrect (Fallacy of denying the hypothesis)
C. Incorrect (Fallacy of affirming the conclusion)
D. Correct (Modus Ponens)

20. A. Let n be an even number. Then $n=2m$ for some m. So $n^2=4m^2=2(2m^2)$. Hence n^2 is an even number.
B. Suppose n^2 is odd. Then n^2 does not contain 2 as a factor. Thus n does not contain 2 as a factor so n is odd. Therefore, if n is even then n^2 is even.
C. Suppose not. Then for some even n, n^2 is odd. Since n is even, let $n=2m$ for some m. Then $4m^2$ is odd for a contradiction. Hence if n is even then n^2 is even.

70. j=Logic is difficult k=Math is difficult l=Many students like logic

1. $j \vee \neg l$
2. $\neg k \to \neg j$

A. $l \to k$
This is valid since applying implication and double negation to 2 gives $k \vee \neg j$. By resolution we then have $\neg l \vee k$ which by implication is $l \to k$.
B. $k \to \neg l$
This is not valid. Consider when j, k, and l are all true.
C. $j \vee k$
This is not valid. Consider when j, k, and l are all false.
D. $k \vee \neg j$
This is valid (see explanation for part A).
E. $l \to (k \vee \neg j)$
This is valid, since by part D the conclusion of the implication must hold.