Discrete Mathematics - Math 55 - Spring 1997 - First Midterm

Instructions: This is a closed book, closed notes, closed calculator, closed computer, closed network, open brain exam. All questions have equal weight. Write all your answers in a blue book. Put your name *and your TA's name and section number* on the blue book.

Question 1. Determine whether or not $(\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q)$ is a tautology. If it is a tautology, prove it without using a truth table. If not, give a counter-example.

Question 2. Classify each of the following functions f as one-to-one, onto, both, or neither. R denotes the set of all real numbers, and R^+ denotes the set of nonnegative real numbers.

2.2 [25%] Let h:R→R^+ be defined by $h(x) = x^2$. Let g:R^+→R be defined by $g(x) = \sqrt{x}$. Finally, let $f = g \circ h$.

2.3 [25%] f:R→R, where $f(x) = 3(x^{1/3}) + 13$.

2.4 [25%] f:D→D, where D is the set {0,1,2,3,4,5,6,7,8,9} and $f(x) =$ the last decimal digit in x^3. (Hint: solving might be faster if you use congruences modulo 10)

2.5 [25%] f:B→C, where B is the set of bits strings of length 10, C is the set of bit strings of length 9, and $f(x)$ is the complement of the last nine bits of x (i.e. 1s are changed to 0s, and 0s are changed to 1s).

Question 3. Find a simple expression in terms of n for $\sum_{k=1}^{n}(k \cdot 2^k)$. (Hint: differentiate $\sum_{k=1}^{n} x^k$.)

Question 4. In the expressions below $\log x$ means $\log_2 x$.

4.1 [20 %] Find the smallest integer n such that $f(x) = 5x^4 + 2x^3 - 6x + 9$ is $O(x^n)$.

4.2 [25 %] Find the smallest integer n such that $f(x) = 16x^2 + x^2(\log x) + 8x(\log(\log x))$ is $O(x^n)$.

4.3 [25 %] Find a simple, small function $g(x)$ such that $f(x) = x^2 \cdot 4^{2x} + x \cdot 6^x$ is $O(g(x))$.

4.4 [30 %] Find a simple, small function $g(x)$ such that

$$f(x) = (5x^4 + 2x^3 - 6x + 9)^5 \cdot (16x^2 + x^2(\log x) + 8x(\log(\log x)))^2 \cdot (x^2 \cdot 4^{2x} + x \cdot 6^x)^3$$

is $O(g(x))$.

Question 5. Use the Euclidean Algorithm to compute $d = \gcd(145, 19)$, and find integers s and t such that $145s + 19t = d$.