

1 Math 228b, Spring 2009: Problem Set 2

Exercise 1 Put an equidistant mesh $a = x_0 < x_1 < \dots < x_N = b$ on the interval $\Omega = [a, b]$ and let ϕ_j be the continuous piecewise-linear nodal basis functions satisfying $\phi_j(x_i) = \delta_{ij}$, with boundary conditions $Au(a) - Bu(b) = 0$. Suppose $N = 2^n$ for some $n \geq 0$ and let θ_{kj} be the hierarchical basis functions for the same solution space for $0 \leq k \leq n$, $0 \leq j < 2^k$ or so. Express each θ_{kj} as (a) scaled and shifted versions of a single function θ and (b) linear combinations of the neighboring finer-level functions $\theta_{k+1,p}$ for $|p - 2j| \leq 1$. Explain what the coefficients in the linear combination mean (hint: smoothing). Carry out the dual computations for the discontinuous piecewise-constant basis τ_j . As a result, write the matrix L_h in the original basis in factored form and explain the significance of the factors.

Exercise 2 Solve Exercise 3.14 of the text.

Exercise 3 Solve Exercise 3.17 of the text.