1 Math 228b, Spring 2009: Problem Set 1

Exercise 1 *Convert the second-order equation*

$$(p(x)y')' + q(x)y = f(x)$$

into a 2×2 *first-order system* $y' = Ay + f$*, where* A *is a* 2×2 *matrix. Discuss the advantages and disadvantages of the two different conversion techniques*

$$u = (y, y')^T$$

and

$$u = (y, py')^T.$$

Exercise 2 *Let* $L : W \to V$ *be a bounded linear operator between Banach spaces* W *and* V*, and define the bilinear form* $a(w, v) = (v, Lw)$ *for* $w \in W$ *and* v *in the dual space* V^* *of* V*. Suppose* a *satisfies the BNB conditions. Show that* L *has a bounded inverse* $L^{-1} : V \to W$*.*

Exercise 3 *Design a finite element method for the well-posed two-point BVP* $u' + Qu = f$*, subject to homogeneous boundary conditions* $Au(a) - Bu(b) = 0$*. Use continuous piecewise quadratic basis functions for the solution space* $W_h \subset W$*, and choose the test function space* $V_h \subset V$ *appropriately. Prove the inf-sup condition for the easy case* $Q = 0$*, and show that for sufficiently smooth solution* u *the method is second-order accurate:*

$$\|u - u_h\|_W \leq Kh^2\|u\|_{W^{3,1}}.$$

Discuss the difficulties involved in the case $Q \neq 0$ *and the tools available for overcoming them.*