2009.10.06.1

Math 128A Problem Set 7 Due 2009.10.16

1. Let $T_n = \begin{bmatrix} 2 & -1 & 0 & & 0 \\ -1 & 3 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & & & -1 \\ 0 & & & -1 & n+1 \end{bmatrix}$ be the

symmetric tridiagonal with $2:n+1$ down the diagonal and $-1$ off the diagonal. Verify that $T_n$ is positive definite and diagonally dominant and that

$$\det(T_n) = (n+1)\det(T_{n-1}) - \det(T_{n-2}) \quad \text{for } n \geq 3.$$

2. Write a program to evaluate $\det(T-\lambda I)$ for any real constant $\lambda$ by QR factorization. Note that $\det(T-\lambda I) = \pm \det(R)$ if $T-\lambda I = QR$. Test on $T_n$ for $n = 1:10$. Optimize your program for tridiagonal matrices.

3. Use bisection and the damped Newton's method with an appropriate choice of the initial interval to find all the eigenvalues of $T_n$ for $n = 1:20$, by finding the zeroes of

$$\varphi(\lambda) = \det(T_n - \lambda I).$$

4. For a general symmetric matrix $A$ design Householder reflections $H(u_1) \ldots H(u_{n-1})$ that reduce $A$ to tridiagonal form

$$T = QAQ^T.$$

Test your method on $RT_nR^T = A_n$ where $R$ is a random orthogonal matrix, for $n = 1:20$. Find the eigenvalues of $A$.