

Math 121A Final Exam, 22 May 2003 ^①

1. Use a Laplace transform to solve
 $y' + y = \pi e^{-t}$, $y(0) = 1$.

2. Solve the heat eqn for a semi-infinite rod insulated at one end:

$$u_t = u_{xx} \quad x > 0, t > 0$$

$$u_x(0, t) = 0 \quad t > 0$$

$$u(x, 0) = e^{-x} \quad x > 0$$

leave your answer as an integral.

3. Evaluate the integral

$$\int_0^{2\pi} \frac{d\theta}{1 - r \sin \theta} \quad r < 1$$

by contour integration.

(2)

4. Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{e^{i\alpha x}}{1+x^4} dx$$

by contour integration. Assume $\alpha \in \mathbb{R}$.

5. Use a Lagrange multiplier to find the closest points to the origin on the surface

$$xyz = 8.$$

6. Suppose the eqns

$$xe^u \cos v + ue^y \sin v = 1$$

$$x^2 + y^2 + u^2 + v^2 = 2$$

determine any two of x, y, u and v in terms of the other two. Evaluate

$$\left(\frac{\partial u}{\partial x}\right)_y \quad \text{and} \quad \left(\frac{\partial u}{\partial v}\right)_x$$

in terms of x, y, u and v .