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$$p = f - w^*(e - v^*pv)w.$$

The elements  $pv$  and  $(e - v^*pv)w$  implement the equivalences

$$p \sim v^*pv \quad \text{and} \quad f - p \sim e - v^*pv.$$

Now it follows from Lemma 1 that  $f \sim e$ .

Von Neumann algebras,  $W^*$ -algebras,  $AW^*$ -algebras and Baer rings are examples of rings with a complete lattice of projections.

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## ON THE FOURIER INVERSION THEOREM FOR $R^1$

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The following is an elementary "noncomputational" proof of the Fourier inversion theorem for tempered distributions on  $R^1$ . The proof does not generalize so easily to  $R^n$ , but the inversion theorem for  $R^n$  can be deduced from that for  $R^1$ .

To get the inversion theorem for tempered distributions it is sufficient, by duality, to have a proof for the space  $\mathfrak{D}$  of *test functions* (i.e. functions  $\phi \in C^\infty$  such that  $\phi^{(m)}(x) = O(|x|^{-N})$  for all  $m$ ,  $N \geq 0$  as  $x \rightarrow \pm \infty$ ). It is also sufficient to consider only the point  $x = 0$ .

**THEOREM.** *There exists a universal constant  $K$  such that*

$$(1) \int_{-\infty}^{\infty} \hat{\phi}(t) dt = K\phi(0) \text{ for all } \phi \in \mathfrak{D}.$$

*The value of  $K$  ( $K = 2\pi$ ) must be determined, as usual, by substituting some particular function  $\phi$ . By linearity, (1) is equivalent to the following:*

$$(2) \phi(0) = 0 \text{ implies } \int_{-\infty}^{\infty} \hat{\phi}(t) dt = 0.$$

**PROOF OF (2).** Since  $\phi(0) = 0$ ,  $\psi(x) \equiv (\phi(x)/x) \in C^\infty$ . By direct computation, since  $\phi(x) = x\psi(x)$ ,  $\hat{\phi}(t) = i(d/dt)\hat{\psi}(t)$ . Then, since  $\hat{\psi}$  is also a test function (direct verification),  $\int_{-\infty}^{\infty} \hat{\phi}(t) dt = i[\hat{\psi}(\infty) - \hat{\psi}(-\infty)] = 0$ . Q.E.D.

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