Suggested Problems for Review
Midterm I. Calculus 1B

Instructor: Zvezdelina Stankova

Note: Many of these problems are harder than the problems on the midterm. Some of these problems will be discussed in Monday's sections. The problems here are only suggested as preparation for the midterm: they are not representative of all types of problems that may appear on the midterm.

Problem 1. Evaluate the following integrals

$$\int_0^2 x^3\sqrt{x^2+4}\,dx;\ \int x^2\sqrt{25-x^2}\,dx;\ \int \frac{3x^2-4x+5}{(x-1)(x^2+1)}dx;\ \int \frac{x^3}{(x+5)^2(x-1)}dx.$$

Problem 2. Use whatever techniques necessary to evaluate the following integrals:

$$\int x^5 e^{-x^3}\,dx;\ \int \frac{\sqrt{1+\ln x}}{x}dx;\ \int \frac{1+\cos x}{\sin x}dx;\ \int \frac{\sqrt{2x-1}}{2x+3}dx;\ \int \frac{x}{1-x^2+\sqrt{1-x^2}}dx\cdot$$

Problem 3. Determine why the following integrals are improper, and whether they converge or diverge. In case they converge, find their value.

$$\int_0^\infty \cos(\ln x)\,dx;\ \int_{-\infty}^\infty \frac{4x}{1+x^2}dx;\ \int_{-\infty}^1 e^{2x}\cos 3x dx;\ \int_1^\infty \frac{\ln x}{x}dx;\ \int_0^4 \frac{1}{x^2+x-6}dx.$$

Problem 4.

(a) Set up an integral for the arc length of the ellipse $\dfrac{x^2}{4}+\dfrac{y^2}{49}=1$. Now revolve the ellipse about the x-axis and set up an integral to evaluate the area of the resulting surface of revolution. Do the same when the ellipse is revolved about the y-axis. No need to evaluate the integrals (unless you feel the urge to discover new formulas. :))

(b) Find the arc lengths of the following curves: $y=\ln(1-x^2),\ 0\le x\le \frac{1}{2};\ y=\frac{1}{3}x^3+\frac{1}{4x},\ 1\le x\le 12.$ Now revolve these curves about the x-axis and find the area of the resulting surfaces of revolution. Do the same when the curves are revolved about the y-axis.

Problem 5*. Prove the following integration formulas:

(a) $\displaystyle\int \frac{dx}{\sqrt{x^2+25}}=\ln|x+\sqrt{x^2+25}|+C$; (b) $\displaystyle\int \frac{dx}{\sqrt{x^2-25}}=\ln|x+\sqrt{x^2-25}|+C$

(c) $\displaystyle\int \sqrt{x^2+25}\,dx=\frac{x}{2}\sqrt{x^2+25}+\frac{25}{2}\ln|x+\sqrt{x^2+25}|+C.$

Note: Compare with Formulas in §7.5-7.6. The point of this exercise is **not** to take the derivative of RHS and check that it equals the integrand on LHS (even though this is a worthy non-trivial exercise on its own), but to use integration methods on LHS to derive RHS. In each case, I would start with trig. substitution, and then follow with whatever methods seem to be needed. In (c), there an alternative sleek solution starting with integration by parts.

Problem 6. Redo your quizzes on approximations of integrals and error bounds.

Problem 7 (bonus). Prove that $\frac{1}{2}(T_n+M_n)=T_{2n}$ for any continuous function $f(x)$ on $[a,b]$.

Problem 8 (bonus). Prove the reduction formula for any integer $n\ge 2$:

$$\int \sin^n x\,dx=-\frac{1}{n}\cos x\sin^{n-1}x+\frac{n-1}{n}\int \sin^{n-2}x\,dx\cdot$$

Use this formula to show that

$$\int_0^{\pi/2}\sin^n x\,dx=\frac{n-1}{n}\int_0^{\pi/2}\sin^{n-2}x\,dx\cdot$$

Apply this formula several times one after, starting with, say, $n=10$ and rolling down to $n=5$ until you see a pattern. Roll down all the way to $n=2$ to show that

$$\int_0^{\pi/2}\sin^{2n}x\,dx=\frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots(2n)}\,\frac{\pi}{2}\cdot$$

Compare with Example 6 and Exercises #43-44 in §7.1.