Calculus 1A: Homework Assignments. Notes and Hints.

Revised 1/19/09

Spring 2009, TT 3:30pm - 5:00pm, Room 105 Stanley Hall

Instructor: Professor Zvezdelina Stankova

HW8. Read §4.2-4.3. Solve and Write Problems:[1]

(1) §4.2: #2,4,6,8,10,12,14,16,18,19,21(a),24,30,31,32.

(a) #2,4: check if the starting and ending values of $f(x)$ are the same, if your function is continuous on the closed interval $[a, b]$, and differentiable on (a, b): this would mean that your function satisfies the conditions of Rolle's Theorem. The numbers satisfying the conclusion of Rolle's Theorem are the x's where $f'(x) = 0$.

(b) #6: find $f'(x)$, set it equal to 0 and show that there are no solutions to $f'(x) = 0$ for $x \in (0, \pi)$; explain in words why this doesn't contradict Rolle's Theorem. (Hint: Are the conditions of Rolle's Theorem satisfied here?)

(c) #10(a): don't use graphing calculators, but just sketch the graphs by hand; for example, you know how to factor the polynomial to get its x-intercepts, and so on. The point of this exercise is the following: in part (a) you see by "naked eye" that the MVT is true and you see the places where the derivative is equal to the total slope; in part (b), on the other hand, you find the actual exact places where the above happens using a formula for the derivative function. Note that the textbook uses the letter c instead of x_0 for the x-coordinate where the tangent slope equals the total slope. Further, study carefully the statement of MVT: the textbook gives you two equivalent algebraic expressions - the first one is as in lecture (tangent slope equals total slope), and the second one is obtained by simple cross-multiplication of both sides by $(b - a)$. It is this second expression that is used in #16, but if it this bothers you, you can safely replace it by the first more familiar expression from lecture.

(d) To contradict a Theorem means to find a "counterexample" to it, i.e., an example in which **all** conditions of the Theorem are satisfied, but the conclusion of the Theorem is false. Well, if you found such an example, then the Theorem itself would be false so it couldn't be called a "theorem". This should tell you that no counterexamples to Theorems exist, and hence no contradictions can arise from Theorems. In the particular exercise #16: the presented example must violate some condition of MVT, so that MVT doesn't apply there. The question is: which condition(s) of MVT are violated in these examples? Of course, don't forget to justify your answers on the odd-numbered problems (instead of only peaking at the answers at the end of the book!)

(e) #18, 19, 21: You need to use Rolle's Theorem. The main idea is that between any two consecutive roots of your function $f(x)$, the derivative must be equal to 0 somewhere; hence, if you have 2 roots, the derivative $f'(x)$ must equal 0 in at least 1 place; if you have 3 roots, the derivative $f'(x)$ must equal 0 in at least 2 places, and so on. Thus, to write a complete solution to a problem saying "Show that there are at most 3 roots." you say "Suppose this is not true, i.e., suppose that there are at least 4 roots." Then, by Rolle's Theorem, between any two consecutive roots $f'(x)$ must be equal to 0 somewhere, i.e., $f'(x) = 0$ in at least 3 places. Find your $f'(x)$ (or reason what type of function it is) and show that this is impossible. Conclude that the supposition is wrong, i.e., (in our particular case above) $f(x)$ has at most 3 roots, etc.

(f) #24: start by making a hypothetical picture of the situation. (Don't use the picture to justify your solution though!) Apply the MVT and see what it says for $f'(x)$. Plug in the data you have and see to what conclusion this leads. Compare with #25 and Ex. 5.

(g) #31: You have to justify your answer in words. Does Cor.2 (in textbook: Cor.7) apply in this situation? Why or why not?

---

[1] I expect that you will be writing a lot of words on this homework. Write down what you do, what you use, how you justify your steps. Read carefully the notes below.

(h) #32: label the LHS and RHS functions by $f(x)$ and $g(x)$. Take the derivatives $f'(x)$ and $g'(x)$ and see that they equal each other. Write that Cor. 2 implies that $f(x)$ and $g(x)$ differ by a constant, i.e., $f(x) = g(x) + c$. Plug now your "favorite" value of x: i.e., a number for which you can easily calculate $f(x)$ and $g(x)$. Check that the two y-values you get are equal, and conclude that then $c = 0$. In other words, $f(x) = g(x)$ for all x so that the wanted identity is proven. Compare with #33 and Ex. 6. Specifically in #32, again recall that the functions listed are arcsin x and arccos x (**not** $1/\sin x$, etc.!!)

(2) §4.3: #2,6,8,12,14,20,22,34,38,40,42.

(a) #2, by the "largest open intervals" on which $f(x)$ is, say, increasing, they mean that you shouldn't unnecessarily break up your intervals; for example, in #1, $f(x)$ is increasing on $[1, 2]$ and also on $[2, 3]$, but that's kind of silly to write because we unnecessarily broke up the big interval $[1, 3]$ into two smaller intervals; so, in #1, I would write that $f(x)$ is decreasing on $[0, 1]$ and $[3, 4]$, and increasing on $[1, 3]$ and $[4, 6]$, etc.

(b) #6, do **not** forget that the given graph is the graph of the **derivative** $f'(x)$. So, don't be looking for intervals where this graph is increasing or decreasing - this is irrelevent; you only have to decide where the graph is under the x-axis, and where it is over the x-axis, because this will tell you where the derivative is negative and where it is positive; use this information to conclude what $f(x)$ does and answer their questions about $f(x)$. Same remark goes here for #8: you are looking at the graph of the **derivative** function $f'(x)$!; part (d): all you have to find is where $f''(x) = 0$, i.e., where the shown derivative function has 0-tangent slope.

(c) #12 is probably going to be hard if you don't tackle it the "easy" way. My suggestion is to simplify both $f'(x)$ and $f''(x)$ until they look like a polynomial over a polynomial. However, don't dare multiply out the denominators, e.g., $(x^2 + 3)^2$ is much better than, say, $x^4 + 6x^2 + 9$! The biggest problem here will come up when you try to figure out where $f'(x)$ is positive/negative, and where $f''(x)$ is positive/negative. For $f'(x)$: there are usually two interesting places to consider - where its numerator is 0, and where its denominator is 0 (the latter will means a vertical asymptote, none in this particular example); these places divide the x-axis into several intervals; the sign of $f'(x)$ will **not** change inside these intervals, hence just pick an $x = a$ in each of these intervals and determine the sign of $f'(x)$ based on what the sign of $f'(a)$ is. As far as the sign of $f''(x)$ is concerned, your job will be easier if you notice that the denominator of $f''(x)$ does not affect its sign at all (why?)

(d) #34,38,40,42: skip the part about checking with a graphing device. Just graph the functions by hand if you can.

I suggest that you write for yourself, very clearly and in an organized manner, all the tests we learned in §4.3. And for each test, draw a couple of examples to illustrate it for yourselves. Thus, there are **4 basic** cases of function behavior:

1. $f(x)$ is increasing **and** concave up on some interval.
2. $f(x)$ is increasing **and** concave down on some interval.
3. $f(x)$ is decreasing **and** concave up on some interval.
4. $f(x)$ is decreasing **and** concave down on some interval.

These cases correspond to the following properties of derivatives:

1. $f'(x) > 0$ **and** $f''(x) > 0$.
2. $f'(x) > 0$ **and** $f''(x) < 0$.
3. $f'(x) < 0$ **and** $f''(x) > 0$.
4. $f'(x) < 0$ **and** $f''(x) < 0$.

The pictures in each case should look like:

1. the right part of a "smile" (draw it! verify it!)
2. the left part of a "frown" (draw it! verify it!)
3. the left part of a "smile" (draw it! verify it!)
4. the right part of a "frown" (draw it! verify it!)