Calculus 1A: Homework Assignments. Notes and Hints.

Revised 1/19/09

Spring 2009, TT 3:30pm - 5:00pm, Room 105 Stanley Hall

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HW15. Read §6.2, §6.5. Solve and Write Problems:

(1) §6.2: #2,4,10,12,14,16,18,32,34,36,42,44,50,52,51,54,56*,58*,68. Make sure that you draw a picture in each and every problem in this HW!

(a) In setting up the integrals to calculate volumes, you can follow the steps below:

(i) Decide with respect to which variable you will be integrating: this depends solely on what kind of axis you are rotating about. If it is a horizontal axis (e.g., x-axis or $y = c$) then you will be integrating wrt variable x. If it is a vertical axis (e.g., y-axis or $x = c$) then you will be integrating wrt variable y.

(ii) Find the beginning and the end of your solid measured on the axis you chose above. These will be your bounds of integration.

(iii) Make sure your functions are written in the "correct" variable: in the variable with respect to which you have decided to integrate. If not, rewrite the functions in the suitable variable. For example, if you are going to integrate wrt variable y, but one function is written as $y = 2x^3 + 4$, then solve for x: $x = ((y - 4)/2)^{1/3}$.

(iv) Next decide what your outer radius will be, and what your inner radius will be. Say, you are rotating a region between $f(y)$ and $g(y)$ about $x = -3$ (say, $f(y)$ is "further away" from $x = -3$ compared to $g(y)$). Then your outer radius will be $(f(y) - (-3))$ and your inner radius will be $(g(y) - (-3))$: you subtract your axis of rotation from your two functions.

(v) Finally, apply the most general formula (the last formula we derived in class) about volumes of solids of revolution. In the example above, rotating about $y = -3$ produces the integral on$[a, b]$ of $\pi((f(y) + 3)^2 - (g(y) + 3)^2)\, dy$. Note that here "$+3$" resulted from subtracting -3 from the two functions. If we had to rotate about $y = 4$, then the outer and inner radius would have been $(f(y) - 4)$ and $(g(y) - 4)$, respectively.

(vi) Of course, if you have only one function to rotate, there is no need to look for "outer" or "inner" radius. You will have only one radius generated by this function and the axis about which you are rotating.

(b) #50: what axis are you rotating about? The function that is being rotated is simply a linear function which passes through points $(R, 0)$ and (r, h); find this linear function and use the formula for solids of revolution (about the y-axis). Make sure you get the bounds of integration correct: what is the lowest and what is the highest point of your solid - these will tell you the beginning and the ending points of your solid on the y-axis. If the letters R, r, h bother you, why don't you try this problem at first with some numbers instead, e.g., $R = 7, r = 3, h = 5$.

(c) #51: integrate wrt the variable y. In other words, find a function which is being rotated about the y-axis and which describes the top "blue" cap of the sphere. The Pytharogus theorem might be helpful here.

(d) #56: make your base circle to be centered at the origin. Calculate the sides of the cross-section squares as your variable x changes between $-r$ and r; use this to calculate the area $S(x)$ of the cross-sections and integrate the function $S(x)$ you obtained.

(e) #58: draw a very good picture of the base: it is between the lines $y = 0$ and $x + y = 1$; next, find the side of the equilateral cross-sections as the variable y moves between 0 and 1; recall that the area of an equilateral triangle with side a is given by $(a^2\sqrt{3})/4$; use this to calculate the area $S(y)$ of the cross-sections and integrate the function $S(y)$ you obtained.

(2) §6.5: #2,4,5,6,8,10,14,18,19. For the die-hards: #20 and #24. Don't forget that you can use Substitution Rule in some of these problems. By the way, #6 can be done with direct integration too - look at your table of basic examples.