Calculus 1A: Homework Assignments. Notes and Hints.

Revised 1/19/09

Spring 2009, TT 3:30pm - 5:00pm, Room 105 Stanley Hall

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HW12. Read §5.1-5.2, Appendix E. Solve and Write Problems:

(1) App. E: #6,10,16,22,30,36.
   (a) #16: the last term gives away a lot! Your dummy variable basically replaces the variable n.
   (b) #22-30: first expand the sum (i.e., write it out without the Σ–notation); after that in #30: use the formula for the sum of the first several natural numbers; if in doubt whether you got the right answer: check by hand for several small values of n.

(2) §5.1: #2,4,18,20,22,26*.
   (a) #18: choose one of R_n, L_n or M_n approximations, partition the interval into n pieces, find the right endpoints (respectively, left endpoints or midpoints), and write R_n (L_n or M_n) as a sum. Finally, in front of what you get, put a limit with n approaching infinity (to fit with the definition of area), and you are done. Compare with #17: there they have gone one step further by writing the sum expression for R_n (L_n or M_n) in Σ notation. You may also try to do this in #18.
   (b) #20: you are basically asked the reverse question to #18. So, in #20, replace the Σ notation by the expanded sum, figure out what the "bases" of your rectangles are and what the heights are, determine the interval $[a, b]$ on which this whole thing is taking place and guess what the function $f(x)$ is. It will be useful to do first #21 and check that their answer coincides with yours.
   (c) #22(a): the easiest would be to find R_n (write them as sums.) #22(b): you are given a formula for sum of the first n cubes. Do **not** attempt to prove this formula, just use it. Mimick what we did for $f(x) = x^2$ in class: factor in front all common stuff from R_n; what is left should be the sum of the first n cubes; replace that using the above formula, and then find the limit of the resulting expression as n approaches infinity; do not forget to use the trick of factoring the highest powers from the top and the bottom!

(3) §5.2: #2,6,8,12,18,20,22,40,42,44,52,54,60,62. A reminder: You must present all your intermediate calculations and wherever appropriate - explain in words what you are doing. If your calculator cannot give answers correct to 6 decimal places, explain this in your HW and give the best approximation which your calculator gives. Note that you cannot use programmable calculators to do the Riemann sums for you - you must write down clearly all the intermediate steps, and leave only the final calculations for your calculator.
   (a) #40: recall how to write $|x - 5|$ as two simpler (linear) functions on two different intervals, and draw the graph of the function before attempting to evaluate the integral.
   (b) #42-44: use the algebraic properties of integrals.
   (c) #52,54,60,62: use the comparison properties of integrals. #52: you have to compare the two functions and show that one is bigger than the other on the given interval $[0, 1]$; thus, start with the inequality of the two functions that you need to show, then square both sides of the inequality and so on keep on simplifying until you end up with something obviously true for all x in $[0, 1]$. Make sure you write all this in your HW. In #60,62: note that $\sin x$ is always between -1 and 1, so replacing $\sin x$ by 1 or -1 might be helpful. In #54: what are the minimum and maximum values of $\cos x$ on the interval $[\pi/6, \pi/4]$? Is $\cos x$ ... maybe ... decreasing on this interval? Draw a picture, decide on the minimum and maximum, and replace $\cos x$ appropriately by them.
   (d) Throughout this HW, you are allowed to use freely (without proof) the formulas for integrals of constant functions, of $f(x) = x$ and $f(x) = x^2$. The last two were discussed in class, and in addition are given in §5.2 #27-28.