Calculus 1A: Homework Assignments. Notes and Hints.

Revised 1/18/04

Spring 2004, TT 8:00am - 9:30pm, Room 2050 Valley LSB

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HW1. Due January 27. Read §1.1-1.3, §2.1-2.2. Solve and Write Problems:

(1) §1.1: #2,10,14,18,26,28,38,42.
(2) §1.2: #2,8,10,12,14,16,18.
(3) §2.1: #2,4,6,8.
(4) §2.2: #6,8,12,14,26,36(a)-(b).

HW2. Due February 3. Read §1.5-1.6, §2.2-2.5. Solve and Write Problems:

(1) §2.3: #2,4,14,20,26,28,32(b)(c),42,48. Justify with LLs!
  (a) #26: put under a common denominator first.
  (b) #28: rewrite as fractions, put under a common denominator.
  (c) #32(b)(c): ignore the graphing part in (a), calculators are OK in (b), but list your table carefully.
  (d) #42: find the LH-limit and the RH-limit.
(2) §2.4: #2,4,6,12,13,16,24.
  (a) #12: use calculator and list your numerical findings.
  (b) #13(a): use the formula πr^2.
(3) §2.5: #4,6,38,42,43.
  (a) #42: for which c are LH- and RH-limits equal?
  (b) #43: justify your answers and include all calculations!

HW3. Due February 10. Read §2.5-2.8. Solve and Write Problems:

(1) §2.5: #44,46,48,52(a),54(a).
  (a) The hypothesis of a theorem includes all conditions of the theorem, i.e. things coming after expressions like "let", "suppose", "assume", "if", "given", "provided", etc. The conclusion of a theorem is what the theorem claims will be true provided all conditions in the hypothesis are satisfied. The conclusion usually comes after expressions like "then", "therefore", "hence", "it follows that", "this forces", "this implies", etc.
  (b) #44: look at your class notes from the IV theorem for examples of functions which do not satisfy some conditions and for them the conclusion of IV theorem fails; and for other examples of functions which do not satisfy some conditions, but the conclusion of IV theorem holds for them nevertheless. In both cases, regardless of the outcome (conclusion is false or it holds) we say that "IV theorem is not applicable" since some conditions in the hypothesis are violated. The only time we can apply a theorem is when all conditions in the hypothesis are satisfied: then the conclusion of the theorem will hold no matter what function you are considering (as long as it satisfies the hypothesis.)
  (c) #46: you have to show that the equation $x^2 = 2$ has a solution. Follow the example from lecture.
(2) §2.6: #4,6,12,16,18,26,52,60.
  (a) #12: factor out x^3 from top and bottom, then cancel, and finally apply limit laws. Be careful in #26: is the problem trying to trick you? #12-26: follow the instructions that they give in the beginning of the problem; do **not** use ϵ/δ definitions here!
  (b) #60: try to sketch the graph of the function when, say, $x > 10$; here you have an M-goal and an N-answer - the textbook calls it (M, N) definition; compare with solution of #12 in §2.4; again you are not asked to find the "first N" from where on $f(x) > M$, but just **some** N from where on $f(x) > M$.

(3) §2.7: #2,6(a)(i),6(b),10,12.

   (a) #2: you are essentially asked to write the definitions of average velocity and of instantaneous velocity; take a look how they did it in #1 in the answers.
   (b) #12: replace the word "screen" by "paper" - no need to use graphing calculators here: plot several points to sketch the graphs.

(4) §2.8: #4,6,8,10(a),18,22.

   (a) #4: use the point-slope formula for the tangent.
   (b) #8,10(a),18: use only what we have studied so far about derivatives (in particular, **no** shortcut formula for finding derivatives from Section 3!)
   (c) #22: no need to find the derivative, just write what $f(x)$ and a are.

HW4. Due February 17. Read §2.9, §3.1-3.2, skim §3.3. Solve and Write Problems:

(1) §2.9: #2,4,6,8,12,28,30,38; Review, pp.178: #38,46.

   (a) #2: "simplifying first" means splitting the fraction along the numerator and simplifying.
   (b) #4: don't forget to give some reason for your answers, e.g. a "sharp corner" indicates no derivative there; or the slopes of the tangents approach 0 as $x \to \infty$, etc.; one good reason is sufficient per graph.
   (c) #6-12: it is best to use different color pens for the original function, for the tangent lines that you will be using, and for the derivative function; first look for any horizontal tangents - these will indicate the zeros of your derivative function; then between every two such tangents draw a few test tangents to see what their slopes are and plot the corresponding points on the graph of the derivative; look also for possible places where the derivative is not defined; if you have trouble with any of these, compare with the answers of #5-11.
   (d) #28-30: you can use any of the two ways (limit formulas with (a, x) or (x, h)) for derivative functions which we discussed in class; make sure you don't forget to include the domain of the original function and the domain of the derivative function. Note that in this Chapter 2, you are asked to find a derivative function for $f(x)$ given by a formula, you can use only the derivative definition methods we have discussed (Hold your horses! **No** differentiation laws from Chapter 3 for these problems!)

(2) §3.1: #12,16,22,24,30,42,46,56.

   (a) So far we have only DLs for addition, subtraction and multiplication by constants, plus base cases for power functions x^n and the exponential function e^x; thus, do **not** use any DLs for multiplication or quotients! Check in particular, #22,24 - rewrite these functions as sums or differences of functions so that you can apply the DLs we know, i.e. in #22: multiply through and simplify **before** applying any DL's; in #24 split the fraction along the numerator and simplify **before** applying any DL's.
   (b) Compare #42 with #41, #46 with #45, #56 with #55. In #42: as usual, replace "screen" by "paper"; use for e an approximation of 2.7, and obviously, you may want to use your ordinary calculator for finding several points on the graph. In #56: calculate the left-side derivative and the right-side derivative at the "troublesome" points, using same technique as for limits of piecewise-defined functions; it may be easier to see what is happening if you first draw the graph of $g(x)$, and use it to find the graph of its derivative $g'(x)$ and determine where $g(x)$ is not differentiable.

(3) §3.2: #2,4,6,12,20,32,42.

   (a) #32, just write first everything out as if you are appyling the DL's to the functions $f(x)$ and $g(x)$, and at the end substitute what you are given about the functions.
   (b) #42: what is the slope of the line $x - 2y = 2$? Solve for y to find this slope, and set your derivative $y'(x)$ =this slope. Find the derivative $y'(x)$, and solve for x the equation $y'(x)$ =this slope.

HW5. Due February 24. Read §3.4-3.6, §3.8-3.9. Solve and Write Problems:

(1) §3.4: #2,10,16,22,30. When a question asks for the equation of a tangent line, first find the derivative of your function, and then use the point slope-formula for the tangent line.

(2) §3.5: #2,4,8,10,20,22,34,46,52*,58.

  (a) #52: an asterik * means that the problem is harder in my opinion, so don't give up easily; in this particular exercise, just get to the point of some equation in terms of $\cos x$ and $\cos 2x$, and leave it at that - it is possible to solve it, but it requires one of the trig. formulas from the begining of the textbook.

  (b) #58: start by writing the formula for the derivative of composite function with the specific given functions, redraw the given graph in your HW and draw the tangents that you are using in your solution; compare with #57.

(3) §3.6: #6,10,18,22*,26,30,48,50. In #22: proceed as if $g(x) = y(x)$ and differentiate both sides with respect to x, then solve for $g'(x)$; finally, substitute in your formula $x = 1$ and $g(1) = 0$.

(4) §3.8: #4,6,12,14,32,40,44. General Note: The only way to get good at applying DLs is to do lots and lots of examples. Thus view this (long) HW as an excellent exercise to help you learn the material, both conceptually and in practice.

HW6. Due March 2. Read §3.5-3.9. Solve and Write Problems:

(1) §3.5: #38,42,48(a):

  (a) #38-42: use the chain rule twice.

  (b) #48(a): we are interested in what happens nearby point $(1, 1)$, i.e. $a = 1 > 0$, so in your formula for y you can safely drop the absolutely value as $|x| = x$; write a word to that effect so that the graders know why you did that.

(2) §3.6: #29,35,42,44.

  (a) #42-44: use the formulas for derivatives of inverse trig. functions given on p.233.

  (b) #29-35: as usual, you must explain your answers and show all calculations.

(3) §3.7: #2,4,8,16,24,34,40,48.

  (a) #2: pick one of the functions and sketch the graph of its derivative to see which other given graph matches it; if no other graph matches it, then your function must be f'''; explain in words what you are doing and how you come up with your choices.

  (b) #4: same problem, except that they have named the first three derivatives by velocity, acceleration and jerk (see textbook for definition of "jerk".)

  (c) #8,16,24: remember that there are no formulas to find directly the higher derivatives of a function; thus, always find the first derivative, then the second derivative from the first derivative, and so on depending on which derivative the problem asks for.

  (d) #34,40: experiment with the first several derivatives until you see a pattern and are fairly certain that the pattern will continue to hold; explain and write all intermediate calculations and cases on which you base your formula; no need to prove your conjectured formula unless you really want to practice the "domino effect" idea (mathematical induction) from p.81.

  (e) #48, obviously, you have to find a formula for the acceleration (after finding a formula for the velocity!), set it equal to 0, solve for t, and then plug this value of t in $s(t)$ and in the velocity $v(t)$.

(4) §3.8: #28,46,47. #47: as usual, you must explain your answer and show all calculations.

(5) §3.9: #30, 32, 40*. Keep in mind that, as with the trig. functions, I don't require you to know $\operatorname{sech} x$ and $\operatorname{csch} x$, so convert everything into $\sinh x$, $\cosh x$, $\tanh x$ and $\coth x$.

HW7. Due March 9. Read §3.11, §4.1. Solve and Write Problems:

(1) §3.11: #4,8,10,18,22,26,36.
(2) Problems Plus for Chapter 3, p.275: #2,12,14. Follow the methods from lecture examples.
(3) §4.1: #4,6,10,14,18,20,22,24,26,32,36,38,40,46.

HW8. Due March 16. Read §4.2-4.3. Solve and Write Problems:[1]

(1) §4.2: #2,4,6,8,10,12,14,16,18,19,21(a),24,30,31,32.
   (a) #2,4: check if the starting and ending values of $f(x)$ are the same, if your function is continuous on the closed interval $[a, b]$, and differentiable on (a, b): this would mean that your function satisfies the conditions of Rolle's Theorem. The numbers satisfying the conclusion of Rolle's Theorem are the x's where $f'(x) = 0$.
   (b) #6: find $f'(x)$, set it equal to 0 and show that there are no solutions to $f'(x) = 0$ for $x \in (0, 2)$; explain in words why this doesn't contradict Rolle's Theorem. (Hint: Are the conditions of Rolle's Theorem satisfied here?)
   (c) #10(a): don't use graphing calculators, but just sketch the graphs by hand; for example, you know how to factor the polynomial to get its x-intercepts, and so on. The point of this exercise is the following: in part (a) you see by "naked eye" that the MVT is true and you see the places where the derivative is equal to the total slope; in part (b), on the other hand, you find the actual exact places where the above happens using a formula for the derivative function. Note that the textbook uses the letter c instead of x_0 for the x-coordinate where the tangent slope equals the total slope. Further, study carefully the statement of MVT on p. 291: the textbook gives you two equivalent algebraic expressions - the first one is as in lecture (tangent slope equals total slope), and the second one is obtained by simple cross-multiplication of both sides by $(b - a)$. It is this second expression that is used in #16, but if it this bothers you, you can safely replace it by the first more familiar expression from lecture.
   (d) To contradict a Theorem means to find a "counterexample" to it, i.e. an example in which **all** conditions of the Theorem are satisfied, but the conclusion of the Theorem is false. Well, if you found such an example, then the Theorem itself would be false so it couldn't be called "theorem". This should tell you that no counterexamples to Theorems exist, and hence no contradictions can arise from Theorems. The particular exercise #16: the presented example must violate some condition of MVT, so that MVT doesn't apply there. The question is: which condition(s) of MVT are violated in these examples? Of course, don't forget to justify your answers on the odd-numbered problems!
   (e) #18, 19, 21: You need to use Rolle's Theorem. The main idea is that between any two consecutive roots of your function $f(x)$, the derivative must be equal to 0 somewhere, hence, if you have 2 roots, the derivative $f'(x)$ must equal 0 in at least 1 place; if you have 3 roots, the derivative $f'(x)$ must equal 0 in at least 2 places, and so on. Thus, to write a complete solution to a problem saying "Show that there are at most 3 roots." you say "Suppose this is not true, i.e. suppose that there are at least 4 roots." Then, by Rolle's Theorem, between any two consecutive roots $f'(x)$ must be equal to 0 somewhere, i.e. $f'(x) = 0$ in at least 3 places. Find your $f'(x)$ (or reason what type of function it is) and show that this is impossible. Conclude that the supposition is wrong, i.e. (in our particular case above) $f(x)$ has at most 3 roots, etc.
   (f) #24: start by making a hypothetical picture of the situation. (Don't use the picture to justify your solution, though!) Apply the MVT and see what it says for $f'(x)$. Plug in the data you have and see to what conclusion this leads. Compare with #25 and Ex. 5 on p.293.
   (g) #31: You have to justify your answer in words. Does Cor.2 (in textbook: Cor.7) apply in this situation? Why or why not?

[1] I expect that you will be writing a lot of words on this homework. Write down what you do, what you use, how you justify your steps. Read carefully the notes below.

(h) #32: label the LHS and RHS functions by $f(x)$ and $g(x)$. Take the derivatives $f'(x)$ and $g'(x)$ and see that they equal each other. Write that Cor. 2 implies that $f(x)$ and $g(x)$ differ by a constant, i.e. $f(x) = g(x) + c$. Plug now your "favorite" value of x: i.e. a number for which you can easily calculate $f(x)$ and $g(x)$. Check that the two y-values you get are equal, and conclude that then $c = 0$. other words, $f(x) = g(x)$ for all x so that the wanted identity is proven. Compare with #33 and Ex. 6 on p.294. Specifically in #32, again recall that the functions listed are arcsin x and arccos x (**not** $1/\sin x$, etc.!!)

(2) §4.3: #2,6,8,12,14,20,22,32,34,38,42.

(a) #2, by the "largest open intervals" on which $f(x)$ is, say, decreasing, they mean that you shouldn't unnecessarily break up your intervals; for example, in #1, $f(x)$ is decreasing on $[6, 7]$ and also on $[7, 8]$, but that's kind of silly to write because we unnecessarily broke up the big interval $[6, 8]$ into two smaller interval; so, in #1, I would write that $f(x)$ is increasing on $[0, 6]$ and $[8, 9]$, and decreasing on $[6, 8]$, etc.

(b) #6, do **not** forget that the given graph is the graph of the **derivative** $f'(x)$. So, don't be looking for intervals where this graph is increasing or decreasing - this is irrelevent; you only have to decide where the graph is under the x-axis, and where it is over the x-axis, because this will tell you where the derivative is positive and where it is negative; use this information to conclude what $f(x)$ does and answer their questions about $f(x)$. Same remark goes here for #8: you are looking at the graph of the **derivative** function $f'(x)$! part (d): all you have to find is where $f''(x) = 0$, i.e. where the shown derivative function has 0-tangent slope.

(c) #14 is probably going to be hard if you don't tackle it the "easy" way. My suggestion is to simplify both $f'(x)$ and $f''(x)$ until they look like a polynomial over a polynomial. However, don't dare multiply out the denominators, e.g. $(x + 1)^3$ is much better than, say, $x^3 + 3x^2 + 3x + 1$! The biggest problem here will come up when you try to figure out where $f'(x)$ is positive/negative, and where $f''(x)$ is positive/negative. For $f'(x)$: there will be two interesting points to consider - where its numerator is 0, and where its denominator is 0 (the latter will means a vertical asymptote); these two points divide the x-axis into several intervals; the sign of $f'(x)$ will **not** change inside these intervals, hence just pick an $x = a$ in each of these intervals and determine the sign of $f(x)$ based on what the sign of $f(a)$ is. As far as the sign of $f''(x)$ is concerned, your job will be easier if you notice that the denominator of $f''(x)$ does not affect its sign at all (why?)

(d) #34,38,42: skip the part about checking with a graphing device. Just graph the functions by hand.

I suggest that you write for yourself, very clearly and in an organized manner, all the tests we learned in §4.3. And for each test, draw a couple of examples to illustrate it for yourselves. Thus, there are **4 basic** cases of function behavior:

1. $f(x)$ is increasing **and** concave up on some interval.
2. $f(x)$ is increasing **and** concave down on some interval.
3. $f(x)$ is decreasing **and** concave up on some interval.
4. $f(x)$ is decreasing **and** concave down on some interval.

These cases correspond to the following properties of derivatives:

1. $f'(x) > 0$ **and** $f''(x) > 0$.
2. $f'(x) > 0$ **and** $f''(x) < 0$.
3. $f'(x) < 0$ **and** $f''(x) > 0$.
4. $f'(x) < 0$ **and** $f''(x) < 0$.

The pictures in each case should look like:

1. the right part of a "smile" (draw it! verify it!)
2. the left part of a "frown" (draw it! verify it!)
3. the left part of a "smile" (draw it! verify it!)
4. the right part of a "frown" (draw it! verify it!)

HW9. Due March 30. Read §4.4-4.5. Solve and Write Problems:

(1) §4.4: #6,10,12,18,22,24,26,28,30,32,40,36,42,44,46,48,50,56. Make sure that you justify on your HW the use of L'Hospital's Rule **before** you apply it, e.g. 0/0 or ∞/∞.
   (a) #6: you may want to do this problem in two different ways: by LH, or by factoring and cancelling.
   (b) #10: be careful!! The solution may turn out to be simpler than you think!
   (c) #18: "$\ln(\ln(\infty))$"="$\ln(\infty)$"$= \infty$; do you need to apply LH in #18 twice, or just once will suffice?
   (d) #22,24,28: you will have to apply LH 2 or 3 times in each problem, so don't give up! As a rule of thumb, every time you apply LH, if possible, simplify/rewrite the resulting quotient before attempting to apply LH again.
   (e) #26 and #40 are nothing special - just review what the relevant functions and derivatives are.
   (f) #30: if you have trouble with m and n, first try the problem when, say, $m = 5$ and $n = 7$, and then repeat your solution with the letters m and n instead of 5 and 7 - these will simply be some constants, whose exact values you don't know.
   (g) #32: one LH will do the job; apply LL immediately to get the final answer.
   (h) #36, #42: be careful – does LH apply here, or is the situation much simpler?
   (i) #44: you have a product indeterminacy, so you have to reciprocate one of the functions to make the whole thing a quotient suitable for LH. Try recoprocating x.
   (j) #46-48, you have to put the fractions under a common denominator **before** you attempt to apply LH.
   (k) #56: you obviously have to rewrite the indeterminate power as an exponential function (compare with Ex.9 on p.313).

(2) §4.5: #10,12,14,30,60,62,64. In many of the problems, note that your function $f(x)$ is a fraction of two polynomials; so when finding its derivatives, do **not** multiply out the denominators; instead, first cancel stuff and factor the numerator as much as possible **before** multiplying out stuff in the numerator. When justifying vertical asymptotes, use the LLs with appropriate $a+$ and $a-$. When justifying horizontal asymptotes, you may use either the factoring trick of highest power of x from top and bottom, or LH - whichever applies, but be careful to check if LH indeed applies!! #54,56,58: when finding slant asymptotes, you may use shortcuts discussed in the class, sesctions and in the handout.

HW10. Due April 6. Read §4.7-4.9. Solve and Write Problems:

(1) §4.7: #2,4,6,8,10,12,16,18,22.
   (a) #2-4 are similar to #3; #6 is similar to Example 1; #10 is similar to Example 2 - don't forget that the box has no top, so don't include the top in the total surface area.
   (b) #16-18 are similar to Example 3 - don't forget to square the distance and find the minimum of this function rather than of the original distance function! (as we did in class in the ship problem.)

(2) §4.8: #2,4,6,8,10,12,14,16,18,20.

(3) §4.9: #2,4,6,12,14,36. #36: find $f'(x)$ and set it to equal 0 in order to find the critical points of $f(x)$. Use Newton's method to estimate the root r of $f'(x)$. Check what is going on with $f''(x)$ to show that $f(r)$ is indeed a local (and global) minimum (as opposed to maximum). Then plug in your estimate for r in $f(x)$ to find this global maximum.

HW11. Due April 13. Read §4.10. Solve and Write Problems:

(1) §4.10: #4,8,10,12,16,18,22,28,32,34,40,42,44,46,48,62,68,74,76,78.
   (a) #8-10: rewrite the powers of x in the usual form x^n to make it easy for integration: this applies both to "roots" in #8 and to reciprocals in #10 (after splitting the fraction).
   (b) #16: please, do **not** use any "quotient rule" for integration - there is **no** such rule! Instead, split the fraction into a sum of 3 fractions and integrate each such fraction separately.

(c) #22 and #40: integrate once to get $f'(x)$, and then integrate another time to get $f(x)$: do **not** forget to add a constant C in the first integration, and then to add another constant D in the second integration (as done in in Example 4). #40: at the end of the day, when the dust settles down and you have found out what $f(x)$ is, use the two given initial conditions to find the exact values of your constants C and D (as in Example 4.)

(d) #68: compare with Example 8. You already know the displacement function $h(t)$ of the ball thrown with inital velocity $48\,\text{ft/s}$. Repeat the same steps to obtain the displacement function $g(t)$ of the ball thrown with initial velocity $24\,\text{ft/s}$. Note that $g(1) = 432\,\text{m}$ because, at time $t = 1\,\text{sec}$, the second ball is at the initial height of $432\,m$. In essence, the question you are being asked - do the two balls pass each other - means: is it true that $h(t) = g(t)$ at some point **before** any of the balls hits the ground? So, solve this equation for t and see if this gives a reasonable practical answer. A good way to visualize the situation is to plot **both** graphs of $h(t)$ and $g(t)$ on the same graph and see if they intersect in a place above the x-axis (this corresponds to the two balls not having hit the ground yet, but being at the same time at the same height.)

HW12. Due April 20. Read §5.1-5.2, Appendix E(p.A41). Solve and Write Problems:

(1) App. E: #6,10,16,22,30,36.

(a) #16: the last term gives away a lot! Your dummy variable basically replaces the variable n.

(b) #22-30: first expand the sum (i.e. write it out without the Σ notation); after that in #30: use the formula for the sum of the first several natural numbers; if in doubt whether you got the right answer: check by hand for several small values of n.

(2) §5.1: #2,4,18,20,22.

(a) #18: choose one of R_n, L_n or M_n approximations, partition the interval into n pieces, find the right endpoints (respectively, left endpoints or midpoints), and write R_n (L_n or M_n) as a sum. Finally, in front of what you get, put a limit with n approaching infinity (to fit with the definition of area), and you are done. Compare with #17: there they have gone one step further by writing the sum expression for R_n (L_n or M_n) in Σ notation. You may also try to do this in #18, but it is strictly speaking not necessary.

(b) #20: you are basically asked the reverse question to #18. So, in #20, replace the Σ notation by the expanded sum, figure out what the "bases" of your rectangles are and what the heights are, determine the interval $[a, b]$ on which this whole thing is taking place and guess what the function $f(x)$ is. It will be useful to do first #21 and check that their answer coincides with yours.

(c) #22(a): the easiest would be to find R_n (write them as sums.) #22(b): you are given a formula for sum of the first n cubes. Do **not** attempt to prove this formula, just use it. Mimick what we did for $f(x) = x^2$ in class: factor in front all common stuff from R_n; what is left should be the sum of the first n cubes; replace that using the above formula, and then find the limit of the resulting expression as n approaches infinity; do not forget to use the trick of factoring the highest powers from the top and the bottom!

(3) §5.2: #2,6,8,12,18,20,34,38,40,47,48,52,54,62. A reminder: You must present all your intermediate calculations and wherever appropriate - explain in words what you are doing. If your calculator cannot give answers correct to 6 decimal places, explain this in your HW and give the best approximation which your calculator gives. Note that you cannot use programmable calculators to do the Riemann sums for you - you must write down clearly all the intermediate steps, and leave only the final calculations for your calculator.

(a) #40: recall how to write $|x - 5|$ as two simpler (linear) functions on two different intervals, and draw the graph of the function before attempting to evaluate the integral.

(b) #47-48: use the algebraic properties of integrals.

(c) #52,54,62: use the comparison properties of integrals. #52: you have to compare the two functions and show that one is bigger than the other on the given interval $[1, 2]$; thus, start with the inequality of the two functions that you need to show, then square both sides of the inequality and so on keep on simplifying until you end up with something obviously true for all x in $[1,2]$. Make sure you write all this in your HW. In #54, 62: note that $\sin x$ is always less than 1, so replacing $\sin x$ by 1 should increase both integrals in 50 and 58. For the left-hand side (LHS) of #54: what is the minimum value of $\sin x$ on the interval $[\pi/6, \pi/2]$? Is $\sin x$ ... maybe ... increasing on this interval? Draw a picture, decide on the minimum, and replace $\sin x$ by this minimum: this should decrease the middle integral in #54.

   (d) Throughout this HW, you are allowed to use freely (without proof) the formulas for integrals of constant functions, of $f(x) = x$ and $f(x) = x^2$. The last two were discussed in class, and in addition are given in #27-28.

HW13. Due April 27. Read §5.3-5.4. Solve and Write Problems:

(1) §5.3: #4,8,10,16,22,26,30,32,40,42,50,52,62,66.
   (a) Be careful with #26!! Are they trying to trick you?
   (b) #30,32,40: use the old trick of writing the function as a power of x: x^a where a could be a fraction, even a negative fraction. In #32-40: if it is difficult for you to find the antiderivative of the whole function, split the integral into a sum of two integrals of simpler functions.
   (c) #50,52: use the "generalized" FTC from lecture (e.g. using upper function $h_2(x)$ and lower function $h_1(x)$). #52: be extra careful since the given function $\cos(u^2)$ is itself a composition of two functions, but this doesn't change the generalized FTC formula if you use it properly.
   (d) #62: recognize this Riemann sum as the integral of some function (which function, and on what interval?), then use FTC to calculate this integral. Compare with #57.

(2) §5.4: #2,4,22,28,38,40,44,48,54,56,58.
   (a) #22-38: multiply out first before attempting to find an antiderivative.
   (b) #48: immitate #47.
   (c) #54,56,58: you have to set up an integral, and then, if the problem is asking, to evaluate this integral.

HW14. Due May 4. Read §5.5-5.6, §6.1. Solve and Write Problems:

(1) §5.5: #2,4,6,8,12,14,18,24,28,32,38,42,52,56,58,64. In trying to guess u, look for part of the **numerator** which looks like the derivative $u'(x)$. For every indefinite integral, you **must** check the answer by differentiation.
   (a) Some hints for substitution: in #24, try $u = 1 + x^{3/2}$. Compare #28 with Example 1, #32 with #27, #52 with Ex. 1, #56 with Ex.8, #58 with Ex.4 (or Ex.1). Note that all of these exercises resemble indeed a **lot** the Examples I refer to above: so, if you have trouble guessing $u(x)$, please read carefully the corresponding Example from the textbook and try to duplicate its idea in the HW problem.
   (b) #52-58: you can proceed the "old" way by first finding the indefinite integral (i.e. an antiderivative) by substitution, and then using FTC I and plugging the bounds of integration into your antiderivative to get a number. Or, you can proceed the "new" way (which will be discussed in workshop): you can use substitution rule and simultaneously change the bounds of integration. Both ways are fine: choose whichever you feel more comfortable with.
   (c) #38,42,64: compare all with Ex. 5. Try the following substitutions: $u = x^3 + 1$ in #38, $u = x^2$ in #42, $u = 1 + 2x$ in #64.

(2) §5.6: #2,4.

(3) §6.1: #2,4,10,14,16,24,45.
   (a) Skip the part about drawing "typical approximating rectangles". However, for each and every problem from 6.1 you **must** provide a clear graph.

(b) #14 (as well as in other problems from 6.1) you will need to find the intersection points of the two given curves; note that in #14 this boils down to solving $x^3 - x = 3x$. A reminder that just giving the answer for #45 (as well as for any other problem) will yield 0 scores. You have to **explain** how you get to your answer and show **all** intermediate calculations.

HW15. Due May 11. Read §6.2, §6.5. Solve and Write Problems:

(1) §6.2: #2,4,10,12,14,16,18,32,34,36,40,44,48,49,54,56. Make sure that you draw a picture in each and every problem in this HW!

(a) In setting up the integrals to calculate volumes, you can follow the steps below:

(i) Decide with respect to which variable you will be integrating: this depends solely on what kind of axis you are rotating about. If it is a horizontal axis (e.g. x-axis or $y = c$) then you will be integrating wrt variable x. If it is a vertical axis (e.g. y-axis or $x = c$) then you will be integrating wrt variable y.

(ii) Find the beginning and the end of your solid measured on the axis you chose above. These will be your bounds of integration.

(iii) Make sure your functions are written in the "correct" variable: in the variable with respect to which you have decided to integrate. If not, rewrite the functions in the suitable variable. For example, if you are going to integrate wrt variable y, but one function is written as $y = 2x^3 + 4$, then solve for x: $x = ((y-4)/2)^{1/3}$.

(iv) Next decide what your outer radius will be, and what your inner radius will be. Say, you are rotating a region between $f(y)$ and $g(y)$ about $x = -3$ (say, $f(y)$ is "further away" from $x = -3$ compared to $g(y)$). Then your outer radius will be $(f(y) - (-3))$ and your inner radius will be $(g(y) - (-3))$: you subtract your axis of rotation from your two functions.

(v) Finally, apply the most general formula (the last formula we derived in class) about volumes of solids of revolution. In the example above, rotating about $y = -3$ produces the integral on$[a, b]$ of $\pi((f(y) + 3)^2 - (g(y) + 3)^2)\, dy$. Note that here "$+3$" resulted from subtracting -3 from the two functions. If we had to rotate about $y = 4$, then the outer and innner radius would have been $(f(y) - 4)$ and $(g(y) - 4)$, respectively.

(vi) Of course, if you have only one function to rotate, there is no need to look for "outer" or "inner" radius. You will have only one radius generated by this function and the axis about which you are rotating.

(b) #40-42: draw a picture and describe in words the solid of revolution as "this-and-this region is being rotated about this-and-this line."

(c) #48: what axis are you rotating about? The function that is being rotated is simply a linear function which passes through points $(R, 0)$ and (r, h); find this linear function and use the formula for solids of revolution (about the y-axis). Make sure you get the bounds of integration correct: what is the lowest and what is the highest point of your solid - these will tell you the beginning and the ending points of your solid on the y-axis. If the letters R, r, h bother you, why don't you try this problem at first with some numbers instead, e.g. $R = 7, r = 3, h = 5$.

(d) #49: integrate wrt the variable y. In other words, find a function which is being rotated about the y-axis and which describes the top "blue" cap of the sphere. The Pytharogus theorem might be helpful here.

(e) #54: make your base circle to be centered at the origin. Calculate the sides of the cross-section squares as your variable x changes between $-r$ and r; use this to calculate the area $S(x)$ of the cross-sections and integrate the function $S(x)$ you obtained.

(f) #56: draw a very good picture of the base: it is between the curves $y = x^2$ and $y = 1$; next, find the side of the equilateral cross-sections as the variable x moves between -1 and 1; recall that the area of an equilateral triangle with side a is given by $(a^2\sqrt{3})/4$; use this to calculate the area $S(x)$ of the cross-sections and integrate the function $S(x)$ you obtained.

(2) §6.5: #2,4,5,6,8,10,14,18,19. Don't forget that you can use Substitution Rule in #5,6,8,19. By the way, #6 can be done with direct integration too - look at your table of basic examples.