# Review Exercises for Midterm 2, Calculus 1A

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## 1. Graph the Original Function from the Graph of its Derivative

**Problem 1.** The graph of the *derivative* $f'(x)$ of a function $f(x)$ is in #31, p.305.

(a) Where is the *original* function $f(x)$ increasing and where is it decreasing? Explain.
(b) Where does $f(x)$ have local minima and maxima? Explain.
(c) Where is $f(x)$ concave up and where is it concave down? Explain.
(d) Where does $f(x)$ have inflection points? Explain.
(e) What happens at $x = 6$? Which of the functions $f(x)$, $f'(x)$ and $f''(x)$ is not defined there? Explain.
(f) Suppose $f(0) = 2$. Sketch a possible graph of the *original* function $f(x)$.

## 2. Graph the Original Function from its Algebraic Formula

**Problem 2.** Consider the function $f(x) = \dfrac{x^2+4}{x}$.

(a) Find all roots of $f(x)$ (if any). Explain.
(b) Find all vertical, horizontal and slant asymptotes of $f(x)$(if any). Explain.
(c) Find all intervals where $f(x)$ is increasing and where it is decreasing. Explain.
(d) Find all local maxima and minima of $f(x)$. Explain.
(e) Find all intervals where $f(x)$ is concave up and where it is concave down. Explain. Does $f(x)$ have inflection points? Why?
(f) Sketch a graph of $f(x)$, including all features you found above.

## 3. Apply MVT or Rolle's Theorem

**Problem 3.** Does there exist a function $f(x)$ such that $f(0) \le -1$, $f(2) = 4$, and $f'(x) \le 2$ for all $x$?

## 4. Apply L'Hospital's Rule

**Problem 4.** Find the following limit. Justify your solution.

(a) $\lim\limits_{x\to 1^+} (x-1)\cdot \tan\dfrac{\pi x}{2}$ (b) $\lim\limits_{x\to 1}\left(\dfrac{1}{\ln x} - \dfrac{1}{(x-1)}\right)$

## 5. Apply Implicit Differentiation

**Problem 5.** Find all points on the curve $x^2y^2 + xy = 2$ where the tangent line slope is $-1$.

## 6. Solve an Optimization Problem

**Problem 6.** The top and the bottom margins of a poster are each 6 cm, and the side margins are each 4 cm. If the area of the printed material on the poster is fixed at 384 cm$^2$, find the dimensions of the poster with smallest area.

## 7. Bonus Problems

**Problem 7*.** The frame for a kite is to be made from six pieces of wood. The four exterior pieces have been cut with the lengths 3 and 4. To maximize the area of the kite, how long should the diagonals be? (Hint: set $x$ to be half of the horizontal diagonal in the textbook picture.)

**Problem 8*.** In an automobile race along a straight road, car $A$ passed car $B$ twice. Prove that at some time during the race their accelerations were equal. (Hint: Apply Rolle's Theorem.)