Review Exercises for Final, Calculus 1A

Instructor: Zvezdelina Stankova

Problem 1. Consider the graph of $f(t)$ in #2, p.402. Let $g(x) = \int_0^x f(t)dt$.

(a) On what intervals is $g(x)$ increasing? Why?
(b) On what intervals is $g(x)$ concave up? Why?
(c) Calculate the 7th midpoint approximation M_7 of $g(7)$.
(d) Calculate the exact value of $g(5)$.

Problem 2. Let $y(x) = \int_{\tan x}^{x^2 - \frac{\pi^2}{16} + 1} \frac{1}{\sqrt{2+t^4}} dt$. Find

(a) $\lim_{x \to \frac{\pi}{4}} y(x)$. Explain.
(b) $y'(\frac{\pi}{4})$. Explain.

Problem 3. A body moves along a straight line with acceleration $a(t) = 2t + 3\ m/sec^2$. The velocity at time $t = 0$ is $v(0) = -4\ m/sec$.

(a) Find the velocity and the speed functions for the time period $-3 \le t \le 10$. Draw their graphs.
(b) Find the average speed for the time period $[-3\,sec, 10\,sec]$.
(c) Find the displacement of the body at time $t = 10\ sec$ relative to the position at $t = -3\ sec$.
(d) Find the total distance travelled for the time interval $[-3\,sec, 10\,sec]$.

Problem 4. Consider the following integrals:

$$A = \int_1^4 2x \ln x\,dx, \quad B = \int_0^3 2x \ln(x+1)\,dx, \quad C = \int_1^4 2(x-1)\ln x\,dx, \quad D = \int_0^9 \ln(\sqrt{x}+1)\,dx.$$

Which of these integrals are equal to each other? Explain. (Substitution Rule will be helpful here.)

Problem 5. Find the numbers a such that the average value of the function $f(x) = (x^2+3)$ on the interval $[a, 1]$ equals 4. Find all $c \in [a, 1]$ for which $f(c) = 4$.

Problem 6. Let $f(x) = x^4 + 1$ and $g(x) = \sqrt[4]{x} + 1$.

(a) Draw the region R determined by the two curves $f(x)$ and $g(x)$, and bound on the left by $x = 0$ and on the right by $x = 2$.
(b) Find the area of the region R.
(c) Set up a formula for the volume of the solid defined by rotating the region R about the x-axis.
(d) Set up a formula for the volume of the solid defined by rotating the region R about $x = -3$.

Problem 7. Calculate $\int \tan x\ \ln(\cos x)\,dx$. (Use Substitution twice.)

Problem 8. If $f'(x)$ is continuous on $[a, b]$, show that $2\int_a^b f(x)f'(x)\,dx = [f(b)]^2 - [f(a)]^2$. (Hint: Substitute.)

Problem 9. Calculate the volume of a solid whose base is the region between the curves $y = x^2$ and $y = 1$, and whose cross-sections perpendicular to the y-axis are squares.

Problem 10. Find $\lim_{h \to 0} \frac{1}{h} \int_2^{2+h} \sqrt{1+t^3}\,dt$. (Use LH and FTC.)