Widely Spaced Vectors

Given two vectors $\mathbf{v}, \mathbf{w} \in \mathbb{C}^n$, we define their inner product as

$$(\mathbf{v}, \mathbf{w}) \equiv \sum_{i=1}^{n} v_i \bar{w}_i,$$

where $\bar{x}$ is the complex conjugate of x. We know how to find vectors $\mathbf{v}_1, ..., \mathbf{v}_n \in \mathbb{C}^n$ so that

$$(\mathbf{v}_i, \mathbf{v}_j) = \delta_{ij} \equiv \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j. \end{cases}$$

Moreover, we know it is impossible to find $n + 1$ such vectors.

What is the smallest $\epsilon > 0$ such that there are unit vectors $\mathbf{v}_1, ..., \mathbf{v}_{n^2} \in \mathbb{C}^n$ satisfying

$$|(\mathbf{v}_i, \mathbf{v}_j)| \leq \epsilon$$

for $i \neq j$?