

Random Reals

Suppose we have a (possibly unfair) coin, where the probability of heads is p and the probability of tails is $1 - p$. If we toss the coin infinitely many times, we obtain the binary expansion of some $x \in [0, 1]$. Given a $y \in [0, 1]$ with a **finite** binary expansion, we may compute

$$P(x \leq y),$$

the probability that the x we obtain is less than or equal to y .

For $n > 0$, let $f_n : [0, 1] \rightarrow [0, 1]$ be the **linear interpolation** of $y \mapsto P(x \leq y)$ for y with binary expansions of length $\leq n$. Let

$$f(x) \equiv^{def} \lim_{n \rightarrow \infty} f_n(x),$$

for each $x \in [0, 1]$.

Is this a definition? What does f look like? Is it continuous? Is it differentiable?